Math 46 Homework 7
Due May 15 at the beginning of class

(1) Page 244 # 4. a) Apply the addition formula, then start guessing eigenfunctions. d) This should be quick.
(2) Page 244 # 7. [Ask if you did not get the eigenvalues and eigenfunctions from #4 c. Each time test if μ is an eigenvalue before proceeding]. This is a nice question: each part gives a different scenario in terms of solvability. For part d please use a right-hand side of $\sin 2x$ instead of $\cos 2x$.
In b and c, you will need the Fourier sine series on $[0, \pi]$,
$$x(\pi - x) = \frac{8}{\pi}\left[\sin x + \frac{\sin 3x}{3^3} + \frac{\sin 5x}{5^3} + \cdots\right]$$
(3) The 1D periodic image function f on $[\pi, \pi]$ is blurred by the periodic kernel defined on $[\pi, \pi]$ by an aperture function $k(s) = 1$ for $|s| < \pi/2$, zero otherwise. i) Give a formula for what blurring does to the Fourier coefficients $a_0, a_1, \ldots, b_1, \ldots$ of f. ii) Give a formula for the deblurred image function in terms of the Fourier coefficients $A_0, A_1, \ldots, B_1, \ldots$ of a measured blurry image. [Reconstruct only the possible coefficients]. iii) Measurement brings an error of 0.01 into all Fourier coefficients of the blurry image. How many coefficients can be reconstructed if the error of any reconstructed coefficient should not exceed 0.3?
(4) Demonstrate that $v(x)$ defined on p.250 indeed solves $Lv = f$, thus the theorem giving the formula for Greens function is correct. Sorry about the algebra, but I care you see how W is canceled from the denominator to leave f. BONUS: Demonstrate that one boundary condition is satisfied.
(5) Page 257 # 1. [View the left hand-side as a differential operator]
(6) Page 257 # 2. You should get an explicit expression for $u(x)$ in terms of f, and state for what f a solution exists. [Hint: use an expansion in eigenfunctions of L, or Thm 4.23]
(7) Page 257 # 5. Appreciate the power of what you have just done. This is a closed-form expression for the solution to arbitrary heat source function in arbitrary conductivity medium! Pretty cool, right?
(8) Page 257 # 7. Sniff around for similar-looking expressions.
(9) Page 257 # 8. You should get an integral operator. Please write two different forms for its kernel: one will involve a sum and the other will not. [Hint: for the sum version use p.224-226 #7 and it is easy]