Math 46 Homework 5
Due May 1 at the beginning of class

(1) Page 150 # 12. Enjoy this beautiful exploration of the mysteries of asymptotic series. $r_n(\lambda)$ is the residual (error in the approximation). Try to be rigorous when it says show that. . . , especially for part c (but do not bother with the full proof by induction for a). For e) please produce a plot of the size of the relative error from the exact answer as a function of n the number of expansion terms summed, in the domain 0 to 20. Make your vertical axis a log scale. This is fascinating. Right? What n is optimal for the approximation? [Hints: for plotting values vs n in Matlab, you should first make a list such as $n = 1 : 20$; then compute everything in terms of this list, e.g. power$(10, n)$ would be the list $10, 10^2, 10^3, ..., 10^{20}$. Note relative error means error as a fraction of the answer. The exact answer is given by the expint command]

(2) Page 214 # 1. (Be careful: the $n = 0$ term will need to be treated specially). Isnt it wild that the function $1x$ has non-zero derivative at the boundary, but the cosines (which have zero derivative there) can approximate it in the mean-square sense?

(3) Page 214 # 3. (Carefully explain the missing details of the proof). This result is important later on, and for every mathematician to know.

(4) Page 215 # 5. You will find even and odd separate, so the Gram-Schmidt will be quick. Then you only need to find c_0 and c_1. Plot and write down the pointwise error for this 2-term approximation. You do not have to compute max pointwise error or mean-square error.

(5) Page 219 # 2. Graph the frequency spectrum means sketch a stick plot of the first few coefficients c_0 , c_1, etc.

(6) Page 225 # 3. If you do not choose to use complex exponentials then you will need to think explicitly about degeneracy of eigenvalues.

(7) Page 225 #4. Unfortunately the energy argument will not work so you will need to try to match boundary conditions for $\lambda < 0$ to show (try to prove) it can or cannot happen. The graphical part is needed since the equation you will get is transcendental.

(8) Page 225 # 6. easy.

(9) a) Write down orthonormal Fourier sine and cosine basis functions on $(-\pi, \pi)$. [Do not forget the constant function].
   b) Use the projection formula to compute coefficients $c_n = (f_n, f)$ which give the function $f(x) = x$ on $(-\pi, \pi)$. [Hint: use symmetry to first discard half the coefficients. Also mess around with http://falstad.com/fourier for fun.]
   c) To what value does this Fourier representation converge to at $x = \pi$?
   d) Apply Parsevals equality to compute $\sum_{n=1}^{\infty} n^{-2}$. Euler first found this value in 1735. [1]

[1] See http://mathworld.wolfram.com/RiemannZetaFunctionZeta2.html