Math 46 Homework 2
Due April 10 at the beginning of class

(1) Page 53 # 6. You will see in part c why this is called a 'pitchfork bifurcation'– please show on your plot.
(2) Page 54 # 10. (quick) This can be a sketch, but label clearly where stable and unstable lie.
(3) Page 100 # 1. This should be quick and easy review of lecture.
(4) Consider the initial value problem

$$u'' - u = \epsilon t u, \qquad u(0) = 1, \quad u'(0) = -1$$

  (a) What is the general solution to the unperturbed problem? What about the particular solution? Under the initial conditions, what is the solution to the unperturbed problem? Does it grow or decay? What do you expect might happen under perturbation– will the solution grow or decay?
  (b) Find the two term perturbation approximation for $0 < \epsilon \ll 1$.
  (c) Using ode45 in MATLAB, find a numerical approximation u for the solution.
  (d) With $\epsilon = 0.04$, plot the numerical solution, u_0, ϵu_1, and u_{approx} on the same axes with $0 \leq t \leq 5$.
  (e) Plot the error function $u - u_{\text{approx}}$ with $0 \leq t \leq 3$. Hint: the error should be $\sim 10^{-3}$ on of the domain. If it is not, check your work!

In the write-up, include all code that you used in the problem (inserted at appropriate points) as well as annotated plots. Be sure to say who you worked with.
(5) Page 100 #3. Be careful: actually proving this is not trivial.
(6) Page 101 #4. This is easy algebra review. Remember to substitute for y.
(7) Page 101 #5 d,g. This should be easy.
(8) Page 101 # 8 a. This ODE could have come from a mass on a nonlinear spring that got weaker with speed squared.
(9) Page 102 # 11. Get the 3-term perturbation approximation. However, you only need to compute t_m and $h_{\max}$ to order ϵ since order ϵ^2 is an algebra nightmare.