Fourier transforms: how to use Table 6.2 backwards

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Note Table 6.2 in Logan's book has many typos. See my Errata, or the back page of last year's final exam

Say you want inverse Fourier transform of $\hat{u}(\xi) = e^{-a|\xi|}$ where a is some constant. This comes up in the FT solution of Laplace's equation in the upper half plane, on p. 393. However, you don't see this function on the ξ (right) side of Table 6.2. But you do see it on the x (left) side. How do you use this? You want to swap roles of x and ξ.

Write out what the table does say, as a forward transform:

$$\int e^{ix\xi} e^{-a|x|} dx = \frac{2a}{a^2 + \xi^2}$$

Divide by 2π and negate ξ:

$$\frac{1}{2\pi} \int e^{-ix\xi} e^{-a|x|} dx = \frac{a}{\pi(a^2 + \xi^2)}$$

Finally swap names of x and ξ:

$$\frac{1}{2\pi} \int e^{-ix\xi} e^{-a|\xi|} d\xi = \frac{a}{\pi(a^2 + x^2)}$$

Thus you have proved the inverse FT you want is $u(x) = \frac{a}{\pi(a^2 + x^2)}$

This trick works since the FT and inverse FT are so similar in form.