MATH 46 WORKSHEET : regular perturbation

4/6/07 Barnett.

Substitute $y(t) = y_0(t) + \varepsilon y_1(t) + \varepsilon^2 y_2(t) \cdots$

into $y' = -y + \varepsilon y^2$ with IC $y(0) = 1$

Collect ε^0 terms :

what IC does $y_0(t)$ satisfy? Solve for it.

Collect ε^1 terms :

What IC does $y_1(t)$ satisfy? [Hint sub series into original IC] Solve for it.

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SOLUTIONS see p. 88-90.

Substitute $y(t) = y_0(t) + \varepsilon y_1(t) + \varepsilon^2 y_2(t) \dots$

into $y' = -y + \varepsilon y^2$ with IC $y(0) = 1$

$$\overbrace{y_0' + \varepsilon y_1' + \varepsilon^2 y_2' + \cdots}= \overbrace{-y_0 - \varepsilon y_1 - \varepsilon^2 y_2 + \cdots} + \overbrace{\varepsilon y_0^2 + 2\varepsilon^2 y_0 y_1 + O(\varepsilon^3) \dots}$$

IC gives

$$y_0(0) + \varepsilon y_1(0) + \varepsilon^2 y_2(0) + \dots = 1.$$

Collect ε^0 terms : $y_0' = -y_0$ ie $y_0' + y_0 = 0$

ie $y_0(t) = Ae^{-t}$

what IC does $y_0(t)$ satisfy? Solve for it.

Taking ε^0 terms in IC gives $y_0(0) = 1$ so $A = 1$

ie $y_0(t) = e^{-t}$

Collect ε^1 terms : $y_1' = -y_1 + y_0^2$

What IC does $y_1(t)$ satisfy? [Hint sub series into original IC] Solve for it.

$$y_1(0) = 0$$

so $y_1' + y_1 = y_0^2 = e^{-2t}$ integrating factor e^t

$$e^t y_1 = \int e^{-t}\,dt + c$$

IC makes $c = +1$

$$y_1 = e^{-t}(-e^{-t} + c) = -e^{-2t} + ce^{-t} = -e^{-2t} + e^{-t}$$