Math 46: Applied Math: Some final practise questions

Focusing on the new material. Work backwards in this list if you want to practise recent stuff. The exam will include material from Midterms 1 and 2 also, but with some preference for new stuff (so split will be about 50% new, 50% Midterm 1 and 2).

Please convince me about any formulae you want added to the back page.

p.7-8: # 1, #4.

p.30-35: # 12.

p.52-54: # 5.

p.121-123: # 1 b.

7.

p.141: # 1

#7.

p.148-150: # 11

p.365-367: #4. Similar to your polar and spherical versions. First answer: what is the formula for rate at which heat flows past a given x value in terms of the function u?

p.381-382: #3 b. (you may use the result of part a)

p.395-398: #6 a.

#12.

A) Use the result (6.46) to prove that a Gaussian convolved with itself gives another Gaussian. How much wider is it than the original?

B) Use the convolution theorem and Ex. 6.30 to find the inverse Fourier transform of $\sin^2(a\xi)/\xi^2$

C) Electric potential satisfies the Laplace equation $u_{xx} + u_{yy} = 0$ in the upper half plane $x \in \mathbb{R}, y > 0$. Use Fourier transforms to solve given boundary data $u(x, 0) = f(x)$. [Hard:] Perform this in the special case $f(x) = H(x)$, corresponding to two abutting electrodes at potentials zero and one.

D) If you want to solve for the electric potential $u(r)$, $r \in (0, \infty)$ due to a radially-symmetric charge density $f(r)$ in 3D, you need to solve Poisson's equation $-\Delta u := -r^{-2}(r^2 u')' = f$. Convert this into Sturm-Liouville form $Lu = h$ giving the new function $h(r)$. The boundary conditions are $u'(0) = 0$ (well-behaved at origin) and $\lim_{r\to\infty} u(r) = 0$ (vanishing at large radii). Find the Green's function. Use this to give the form of the electric potential inside and outside a spherical shell of charge $f(r) = \delta(r-a)$. Do the same for a uniform ball of charge $f(r) = r^2$ for $r < a$, zero otherwise. (Note that this is uniform since f is the source density per unit radius, not per unit volume).

E) Use a Fourier transform to solve the 1D growth-diffusion equation

$$u_t = Du_{xx} + \mu u$$

with general initial conditions $u(x, 0) = f(x)$ on $\mathbb{R}$. Find a formula for the solution involving erf, for the case $f(x) = 1$ for $|x| < a$, zero otherwise. For what values of μ does $u(x, t)$ diverge as $t \to \infty$, pointwise for all x?

Some answers

C) Same as book, Example 6.35. For the heaviside function BCs, the solution is $u(x,y) = 1 - \frac{1}{\pi}\tan^{-1}(y/x)$

D) $g(r,\xi) = \begin{cases} \xi^{-1}, & r < \xi \\ r^{-1}, & r > \xi \end{cases}$

For the shell, the potential is $u(r) = g(r,a)$. For the ball it is $u(r) = \begin{cases} a^2/2 - r^2/6, & r < a \\ a^3/3r, & r > a \end{cases}$