Math 46: Applied Math: Homework 2

due Wed Apr 15 ... but best if do relevant questions after each lecture

From p.100 #2 onwards, which is the meat of the problem set, always check how many terms the question asks for, *e.g.* $y_0 + \varepsilon y_1$ is 2-term. You'll also need to allow time to get Matlab to produce the right plots.

p.40-44: #5. A warm-up question (no pun intended). Write your answer to b in the following way: move both exponential terms into the integral to simplify to a single exponential. Please interpret as a weighted average of $\theta(t)$. This convolution result is called *Duhamel's principle*.

p.52-54: #6. You will see in c why this is called a 'pitchfork bifurcation'—please show the pitchfork on your plot.

#10. (quick). This can be a sketch, but label clearly where stable and unstable lie.

p.67-68: #2. For this you'll need to look up your phase plane linear stability from Math 23. The point is to see that stability can suddenly change with a parameter. Try to visualize how the two eigenvalues move in the complex plane as b varies. Note you don't need a full solution for each case of b, just discussion of behavior (type of critical point), including the equal-roots case.

p.100-104: #1. This is a quick and easy review of Lecture 2 (see the Errata in the formula).

#2. This is a lovely example. Please leave enough time to get it right and produce the plots—you will love it when it works. First ask yourself, is the unperturbed ODE oscillatory or decaying/growing? You will find the ICs given cause the unperturbed solution to be special (how?), and the perturbation messes this up in a dramatic way. Please don't bother finding, or plotting, the Taylor series. Instead produce the following two plots at $\varepsilon = 0.04$:

- compare $u(t)$, $u_0(t)$, $\varepsilon u_1(t)$, and $u_a(t)$ on the same axes in the domain $t \in [0, 5]$
- show error $E(\varepsilon, t) := u_a(t) - u(t)$ in the domain $t \in [0, 3]$, making sure your axes illustrate its size

You should find the error is very small, staying much smaller than 10^{-3} in most of the latter domain. If you don't find this, you'll need to debug your algebra! [*e.g.* make sure $u_1(t)$ satisfies the correct ICs]

#3. Be careful: actually proving this isn't trivial.

#4 (easy algebra review; remember to substitute for y!)

#5 d, g (should be easy).

#8. a. This ODE could have come from a mass on a nonlinear spring that got weaker with speed squared.

#11. (connects to the planet-projectile ODE scaling problem from Lecture 3). Getting the 3rd term involves some high powers of t; do not be alarmed. However, only compute t_m and h_{max} to order ε since order ε^2 is an algebra nightmare.