Math 46: X hour of 5/10/07

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We used Section 4.3.3, particularly Thms 4.12 and 4.13, to determine if the following had a solution, and then solve them. We made use of (4.31) a lot to get $u(x)$ once the $\mathbf{c}$ vector was found.

Let K operator have kernel $k(x, y) = \sin x \sin y$.

Then A is 1-by-1 matrix with entry $\pi/2$. Spectrum of K is then $\pi/2$ (multiplicity 1, eigenfunction $\sin x$), and 0 (infinite multiplicity, eigenspace all functions orthog to $\{\beta_j\}$ that is $\sin x$)

Solve the following:

1. $Ku - u = \sin 2x$
2. $Ku - u = x$
   (We used Maple to get the Fourier coefficient)
3. $Ku - 3\sin 2x$
4. $Ku = 3\sin x$.

Answer key:

1. $c_1 = 0$ so $u = -\sin 2x$
2. $c_1 = \frac{\pi}{1-\pi/2}$ so $u = \frac{\pi}{1-\pi/2}\sin x \; - \; x$
3. no solution
4. $u = \frac{6}{\pi}\sin x$ + (any function orthogonal to $\sin x$). Infinitely-nonunique solution.