Math 46: Applied Math: Homework 7—modified

due Wed May 14 ... but best if do relevant questions after each lecture

In the first question you'll need the Fourier sine series on $[0, \pi]$,

$$x(\pi - x) = \frac{8}{\pi}\left[\sin x + \frac{\sin 3x}{3^3} + \frac{\sin 5x}{5^3} + \cdots\right]$$

How did I quickly work out the coefficients? Via these Maple commands—try it!

```
assume(n,integer);
assume(n,odd);                                  # by symmetry you know only odd sines contribute
int(sin(n*x)*x*(Pi-x), x=0..Pi) / int(sin(n*x)**2, x=0..Pi);      # coeffs (f,f_n)/||f_n||^2
```

p.243-247: #7. [ask if you didn't get the eigenvalues and eigenfunctions from #4 c. Please test if μ is an eigenvalue before proceeding]. This is a nice question: each part gives a different scenario in terms of solvability. For part d please use a right-hand side of $\sin 2x$ instead of $\cos 2x$. Congratulations, you've now solved your first symmetric Fredholm integral equations, infinite-dimensional problem!

A) The 1D image function f on $[-\pi, \pi]$ is blurred by the periodic kernel defined on $[-\pi, \pi]$ by an 'aperture function' $k(s) = 1$ for $|s| < \pi/2$, zero otherwise. i) Give a formula for what blurring does to the Fourier coefficients $a_0, a_1, \ldots, b_1, \ldots$ of f. ii) Give a formula for the deblurred image function in terms of the Fourier coefficients $A_0, A_1, \ldots, B_1, \ldots$ of a measured blurry image. [reconstruct only the possible coefficients]. iii) Measurement brings an error of 0.01 into all Fourier coefficients of the blurry image. How many coefficients can be reconstructed if the error of any reconstructed coefficient should not exceed 0.3?

B) Demonstrate that $v(x)$ defined on p.250 indeed solves $Lv = f$, thus the theorem giving the formula for Green's function is correct. Sorry about the algebra; but I care you see how W is cancelled from the denominator to leave f. Check one BC is satisfied too.

p.257-258: #1. [view the LHS as a differential operator]

#2. You should get an explicit expression for $u(x)$ in terms of f. [Hint: use an expansion in eigenfunctions of L]

#5. Appreciate the power of what you've just done: a closed-form expression for the solution to arbitrary heat source function in arbitrary conductivity medium!

#7.

#8. You should get an integral operator. Please write *two* different forms for its kernel: one will involve a sum and the other won't. [Hint: for the sum version use p.224-226 #7 and it's easy]