Math 46: Applied Math: Homework 6

due Wed May 7 ... but best if do relevant questions after each lecture

Trying the questions in the strange order I give will be easier.

SLP eigenproblems

p.224-226: #6. easy.

#7. Amazing what comes out of an innocent little equation.

#8.

Volterra equations, transforming back and forth to IVPs

p.243-247: #9. [Hint: bring out e^t, turn it into an ODE which you should state, solve with ICs]

#6. [you'll need Leibniz formula from #1. Don't forget the ICs!]

#24. [Hint: use Lemma 4.9]. Make sure to state f and K.

#8. Be careful about using integration variables distinct from limits. You'll get a polynomial in t.

Fredholm with degenerate kernel, equivalent to matrix problems

#2. This is to show you the linear algebra analogy of what happens for Fredholm integral equations. However there are typos: c) should read $A\mathbf{x} - 5\mathbf{x} = (1, -1/2, 0)^{\mathrm{T}}$ and d) $A\mathbf{x} - 5\mathbf{x} = (1, 4, 0)^{\mathrm{T}}$. [Hint: you will find using the unnormalised eigenvectors easier as the $\mathbf{e}_i$ in p.228-229. Quote your coefficients c_i, $i = 1, 2, 3$, which I'll check–you don't even need to write the solution $\mathbf{u}$. c) will be very quick]

#13. a. The simplest possible degenerate kernel, but you could just integrate to solve. State the special value of λ and for this value give condition(s) on f such that a solution exists.

c. Now you'll actually need to write down functions $\alpha_1(x)$, $\beta_1(x)$.

A. Find the eigenvalues and eigenspaces of the integral operator

$$(Ku)(x) = \int_0^\pi \sin x \, \sin y u(y) dy.$$

B. Find the spectrum and eigenfunctions of the integral operator

$$(Ku)(x) = \int_0^1 (1 - 5x^2y^2) u(y) dy.$$

Is $Ku - u = f$ soluble given the function $f(x) = x$? If so, find the solution $u(x)$. Is it unique?

Fredholm with continuous kernel, the joy of infinite dimensions

#4. c. Keep taking the derivative and cancelling lots of stuff until you've transformed to a SLP!

d.

a. Harder. [Hint: addition formula, then start guessing eigenfunctions.]