# Math 46: Applied Math: Some final practise questions

Focusing on the new material. Work backwards in this list if you want to practise recent stuff. The exam will include material from Midterms 1 and 2 also, but with some preference for new stuff (so split will be about 45-50% new, 50-55% Midterm 1 and 2).

You may bring a single side of letter paper notes. You will be provided with a table of Fourier transforms similar to Table 6.2, and the same other formulae as Midterm 2. So things like Green's identities and defn of Fourier you should write down or remember.

**p.7-8**: # 1.

**p.30-35**: # 12.

**p.121-123**: # 1 b.

# 7.

**p.141**: # 1

#7.

**p.148-150**: # 11

**p.365-367**: #4. First answer: what is the formula for rate at which heat flows past a given $x$ value in terms of the function $u$?

**p.381-382**: #3 b. (you may use the result of part a)

**p.395-398**: #6 a.

#12.

A) Use the result (6.46) to prove that a Gaussian convolved with itself gives another Gaussian. How much wider is it than the original?

B) Use the convolution theorem and Ex. 6.30 to find the inverse Fourier transform of $\sin^2(a\xi)/\xi^2$

C) Electric potential satisfies the Laplace equation $u_{xx} + u_{yy} = 0$ in the upper half plane $x \in \mathbb{R}, y > 0$. Use Fourier transforms to solve given boundary data $u(x, 0) = f(x)$. [Hard:] Perform this in the special case $f(x) = H(x)$, corresponding to two abutting electrodes at potentials zero and one.

D) If you want to solve for the electric potential $u(r)$, $r \in (0, \infty)$ due to a radially-symmetric charge density $f(r)$ in 3D, you need to solve Poisson's equation $-\Delta u := -r^{-2}(r^2u')' = f$. Convert this into Sturm-Liouville form $Lu = h$ giving the new function $h(r)$. The boundary conditions are $u'(0) = 0$ (well-behaved at origin) and $\lim_{r\to\infty} u(r) = 0$ (vanishing at large radii). Find the Green's function. Use this to give the form of the electric potential inside and outside a spherical shell of charge $f(r) = \delta(r-a)$. Do the same for a uniform ball of charge $f(r) = 1$ for $r < a$, zero otherwise.

E) Use a Fourier transform to solve the 1D growth-diffusion equation

$$u_t = Du_{xx} + \mu u$$

with general initial conditions $u(x, 0) = f(x)$ on $\mathbb{R}$. Find a formula for the solution involving erf, for the case $f(x) = 1$ for $|x| < a$, zero otherwise. For what values of $\mu$ does $u(x, t)$ diverge as $t \to \infty$, pointwise for all $x$?