Math 46: Applied Math: Homework 8

due Wed May 23 ... but best if do relevant questions after each lecture

p.345-346: #2. a. [Hint: get the general solution with y held const]

d. [If you're ever unsure you have the right solution, substitute back into the PDE to check it works!]

e.

#3. You'll need to think how to satisfy the BC and IC, check it does.

p.365-367: #1. Instead use the transformation $w = e^{u/k}$. [Things should cancel if you do chain rule correctly. Then see Example 6.6, don't forget to transform back!]

#3.

#5. Here you derive that the radial part of the laplace operator in 3D cylindrical (or 2D polar) coordinates is $\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r}\cdot)$

#11. Note that z is the only dimensionless parameter you can make from x, k and t. The situation is sticking an initially uniform-temperature rod against a hot oven at constant temperature; also it gives the probability of having hit the left wall in a random walk (see 6.2.4 for random walk connection).

p.371-374: #5. easy

#6. again, easy, adapting a method from 1D. In fact $-\Delta$ is a 'positive operator'. Note the λ values would be eigenvalues of the Laplacian.

#10. This is much more of a modeling question, something to keep practised at. Coming up with the BCs is a fun part.

p.381-382: (You will be able to do these even though I won't have lectured from 6.4. I chose them as good review).

#1. a, c. This is a good review of the separation-of-variables technique you should have done in Math 23. Dig up your notes.

#3. a. The L given is the higher-dimensional analogous form to a Sturm-Liouville operator. Note that if the boundary term vanishes (*e.g.* homogeneous BCs) you've proved *self-adjointness* of L, *i.e.* $(u, Lv) = (Lu, v)$ for all u, v.