Math 46: Applied Math: Homework 7—modified

due Wed May 16 ... but best if do relevant questions after each lecture

In the first question you'll need the Fourier sine series on $[0, \pi]$,

$$x(\pi - x) = \frac{8}{\pi}\left[\sin x + \frac{\sin 3x}{3^3} + \frac{\sin 5x}{5^3} + \cdots\right]$$

How did I quickly work out the coefficients? Via these Maple commands—try it!

```
assume(n,integer);
assume(n,odd);                                    # by symmetry you know only odd sines contribute
int(sin(n*x)*x*(Pi-x), x=0..Pi) / int(sin(n*x)**2, x=0..Pi);      # coeffs (f,f_n)/||f_n||^2
```

p.243-247: #7. [ask if you didn't get the eigenvalues and eigenfunctions from #4 c. Please test if μ is an eigenvalue before proceeding]. This is a nice question; each part gives a different scenario in terms of solvability. Congratulations, you've now solved your first symmetric Fredholm integral equation, an infinite-dimensional problem!

A) Demonstrate that $v(x)$ defined on p.250 indeed solves $Lv = f$, thus the theorem giving the formula for Green's function is correct.

p.257-258: #1. [view the LHS as a differential operator]

#2. You should get an explicit expression for $u(x)$ in terms of f. [Hint: use an expansion in eigenfunctions of L]

#3. [Hint: you can solve as is although your solution may involve an integral. Then to get the $p(\xi)$ function you'll need to transform to S-L form using an integrating factor]

#5. Appreciate the power of what you've just done: a closed-form expression for the solution to arbitrary heat source function in arbitrary conductivity!

#7.

#8. [Hint: use p.224-226 #7 and it's very easy]

p.345-346: #1. Note this is 1D equivalent of the heat spreading function you studied in 3D in the dimensional analysis worksheet.

#6.