The Exam

- The preliminary exam is Friday.
- The exam is in two parts. The "in-class" part taken during our lecture period and a short "take-home" part due Monday prior to the start of class.
- Just as in the sample exam, the in-class part is primarily objective concentrating on definitions, statements of results, and what *I believe to be* straightforward computations or short arguments.
- The exam covers everything we did through and including section 3.5 in the text. There is nothing from Chapter 4.
- Because of the exam, this week, our homework is due on Wednesday.

Last Time

Definition

Suppose that γ is a directed smooth curve with admissible parameterization $z : [a, b] \to \mathbb{C}$. If f is continuous on γ, then the contour integral of f along γ is

$$\int_\gamma f(z)\,dz := \int_a^b f(z(t))z'(t)\,dt. \qquad (1)$$

If $\Gamma = \gamma_1 + \cdots + \gamma_n$ and f is continuous on Γ, then the contour integral of f along Γ is

$$\int_\Gamma f(z)\,dz := \sum_{k=1}^{n} \int_{\gamma_k} f(z)\,dz.$$

Remark

The value of (1) is independent of our choice of an admissible parameterization for γ.

Key Theorem

Theorem

Let C_r be the positively oriented circle of radius r centered at z_0. Then for any $n \in \mathbf{Z}$,

$$\int_{C_r} (z - z_0)^n \, dz = \begin{cases} 2\pi i & \text{if } n = -1, \text{ and} \\ 0 & \text{if } n \neq -1. \end{cases}$$

Estimates

Proposition

If $z : [a, b] \to \mathbb{C}$ is continuous, then

$$\left| \int_a^b z(t)\, dt \right| \leq \int_a^b |z(t)|\, dt.$$

Theorem

Suppose that $|f(z)| \leq M$ for all z in a contour Γ. Then

$$\left| int_\Gamma f(z)\, dz \right| \leq M\ell(\Gamma).$$

Observation

- Suppose that f is continuous on the contour $\Gamma = \gamma_1 + \cdots + \gamma_n$ and that $z : [a, b] \to \mathbb{C}$ is an admissible parameterization of Γ.
- That means there is a partition $\{ a = \tau_0 < \tau_1 < \cdots < t_n = b \}$ of $[a, b]$ such that the restriction of z to $[\tau_{k-1}, \tau_k]$ is an admissible paramterization of γ_k for $k = 1, \dots, n$.
- Thus

$$\begin{aligned}\int_\Gamma f(z)\,dz = \sum_{k=1}^n \int_{\gamma_k} f(z)\,dz = \sum_{k=1}^n \int_{\tau_{k-1}}^{\tau_k} f(z(t))z'(t)\,dt \\ = \int_a^b f(z(t))z'(t)\,dt.\end{aligned}$$

- If Γ consists of a single point z_0, then we define $\int_\Gamma f(z)\,dz$ to be zero. This is consistent with saying that the constant function $z : [a, b] \to C$ given by $z(t) = z_0$ is an admissible parameterization of Γ and "evaluating"

$$\int_\Gamma f(z)\,dz = \int_a^b f(z(t))z'(t)\,dt.$$