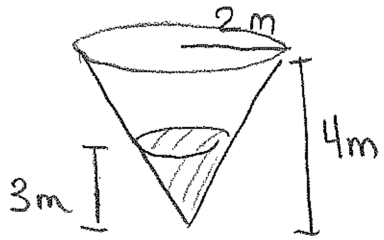
Related Rates

(1) A water tank has the shape of an inverted circular cone with base radius 2 m and height 4m. If water is being pumped into the tank at a rate of 2 m^3/min, find the rate at which the water level is rising when the water is 3 m deep.



Volume: $\frac{1}{3}\pi r^2 h$ have: $\frac{dV}{dt}$)

want: $\frac{dh}{dt}$ when $h = 3$ m

$V = \frac{1}{3}\pi r^2 h$: we need to eliminate r

Similar triangles: $\frac{r}{h} = \frac{2}{4} = \frac{1}{2}$

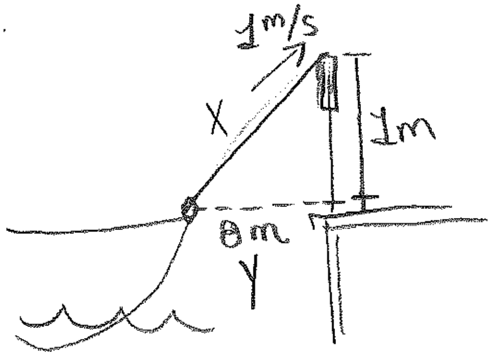
so $r = h/2$

$V = \frac{1}{3}\pi \frac{h^2}{4} \cdot h$

$V = \frac{\pi}{12} h^3$

$\frac{dV}{dt} = \frac{\pi}{12} \cdot 3h^2 \cdot \frac{dh}{dt}$ $\quad 2 = \frac{\pi}{4} \cdot 9 \cdot \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{8}{9\pi}$

(2) A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 1 m higher than the bow of the boat. If the rope is pulled in at a rate of 1 m/s how fast is the boat approaching the dock when it is 8 m from the dock?



when the boat is 8 m from the dock,

$X^2 = 8^2 + 1^2$

$X = \sqrt{65}$

$X^2 = Y^2 + 1^2$

$\frac{d}{dt} X^2 = \frac{d}{dt}(y^2 + 1)$

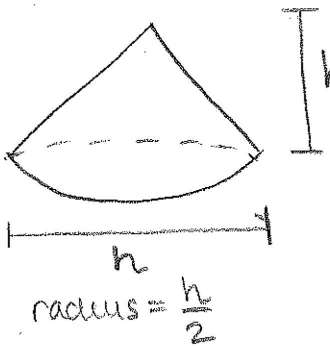
$2x \cdot \frac{dx}{dt} = 2y \cdot \frac{dy}{dt}$

$2\sqrt{65} \cdot 1 = 2(8)\frac{dy}{dt}$

$\frac{\sqrt{65}}{8} = \frac{dy}{dt}$

$\underbrace{1.014 \text{ m/s}}_{\text{how fast boat is approaching}} = \frac{dy}{dt}$

(3) Gravel is being dumped from a conveyor belt at a rate of 30 ft^3/min and its coarseness is such that it forms a pile in the precise shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 10 ft high?



$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 h$$

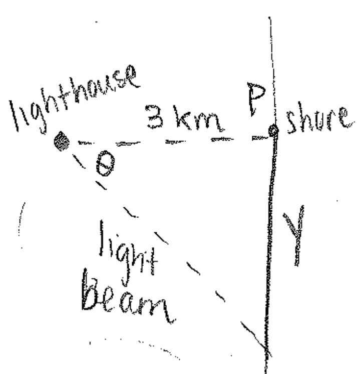
$$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{\pi}{12}h^3\right)$$

$$\frac{dV}{dt} = \frac{\pi}{4}h^2 \cdot \frac{dh}{dt}$$

$$30 = \frac{\pi}{4}(10)^2 \frac{dh}{dt}$$

$$\frac{12}{10\pi} = \boxed{\frac{6}{5\pi}\ \text{ft/min} = \frac{dh}{dt}}$$

(4) A lighthouse is on a small island 3 km away from the nearest point P on the straight shoreline and its light turns four revolutions per minute. How fast is the beam of light moving along the shoreline when it is 1 km from P? (Hint: four revolutions per minute means that the relevant angle is changing at $4 \cdot 2\pi$ radians per minute).



What is $\frac{dy}{dt}$?

relate θ and y:

$$\tan\theta = y/3$$

$$\frac{d}{dt}(\tan\theta) = \frac{d}{dt}(y/3)$$

$$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{3}\frac{dy}{dt}$$

when $y=1$

$$\sec^2\theta = \left(\frac{hyp}{adj}\right)^2 = \left(\frac{\sqrt{9+1}}{3}\right)^2 = \frac{10}{9}$$

$$\left(\frac{10}{9}\right)(8\pi) = \frac{1}{3}\frac{dy}{dt}$$

$$\boxed{\frac{80\pi}{3}\ \text{km/min} = \frac{dy}{dt}}$$