Position, Velocity, Acceleration Word Problems:

1. An object dropped from a cliff has acceleration a = 9.8 m/s² under the influence of gravity. What is the function s(t) that models its ~~height~~ at time t? distance fallen

NOTE: This 9.8 m/s² acceleration is downwards

$a(t) = -9.8\ m/s^2 \leftarrow$ given

$v(t) = \int a(t)\,dt = \int -9.8\,dt$

$= -9.8t + C$ we need to solve for C. Notice $v(0)=0$ (velocity is 0 at time 0)

$v(0) = -9.8(0) + C \Rightarrow C = 0$

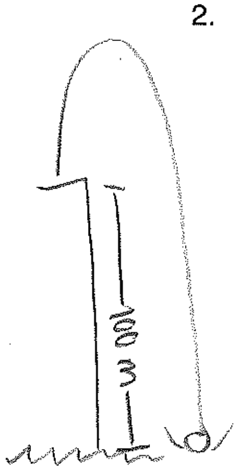
So $v(t) = -9.8t$

$s(t) = \int v(t)\,dt = \int (-9.8t)\,dt = \frac{-9.8}{2}t^2 + D$ solve for D, $s(0)=0$

$s(0) = \frac{-9.8}{2}(0)^2 + D$

$0 = D$ $s(t) = \frac{-9.8}{2}t^2 = \boxed{-4.9t^2}$

2. Suppose that a baseball is thrown upward from the roof of a 100 meter high building. It hits the street below eight seconds later. What was the initial velocity of the baseball, and how high did it rise above the street before beginning its descent? It will change direction at it's highest pt when $v(t)=0$



$a(t) = -9.8\ m/s^2$

$v(t) = \int a(t)\,dt = \int (-9.8)\,dt = -9.8t + C$

$s(t) = \int (-9.8t + C)\,dt = -4.9t^2 + Ct + D$

at $t=0$, $s(0)=100$ $s(0) = -4.9(0)^2 + C(0) + D = D = 100$

at $t=8$, $s(8)=0$ $s(8) = -4.9(8)^2 + C(8) + 100$

$C = 26.7$

$v(t) = -9.8t + 26.7$ $s(t) = -4.9t^2 + 26.7t + 100$

$v(0) = 26.7$ initial velocity. $v(t) = 0 = -9.8t + 26.7 \Leftrightarrow t = 2.72$ when ball is at highest point

3. (Harder) A car braked with constant deceleration of 16 ft/s², producing skid marks measuring 200ft before coming to a stop. How fast was the car traveling when the brakes were first applied?

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$s(2.72)$

$= -4.9(2.72)^2 + 26.7(2.72) + 100$

$= 136.37$ m

highest point

$a(t) = -16$ (deceleration so negative)

$v(t) = -16t + C$

$p(t) = -8t^2 + Ct + D$

we will let position be distance from when brakes were first applied.

so $p(0) = 0 \Rightarrow D = 0$

let t_0 be the time when car completely halts.

$v(t_0) = 0$ when $p(t_0) = 200$

the car has velocity 0 after it has travelled 200 ft from brakes being applied.

$0 = -16t_0 + C \Rightarrow 16t_0 = C$

$200 = -8t_0^2 + Ct_0 \Rightarrow 200 = -8t_0^2 + 16t_0^2$

$200 = 8t_0^2$

$\sqrt{\frac{200}{8}} = t_0 = 5$ s

at 5s, the car stops.

So $v(5) = -16(5) + C = 0$

$C = 80$

$v(t) = -16t + 80$

$v(0) = \underline{80}$

the car was travelling 80 ft/s when it started braking.

Incidentally:

$80 \frac{ft}{s} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} \cdot \frac{3600 \text{ s}}{1 \text{ hr}} = 54.55$ mph