Practice with Limits

Evaluate the following limits

(1) $\lim_{x\to 1}(6x^2 - 4x + 3)$

$= 6(1)^2 - 4(1) + 3$

$= 5$

(2) $\lim_{x\to 7} \frac{x^2 - 49}{x - 7} = \lim_{x\to 7} \frac{\cancel{(x-7)}(x+7)}{\cancel{x-7}}$

$= \lim_{x\to 7} x + 7 = 14$

(3) $\lim_{x\to 2} \frac{x^2 - 6x + 8}{x - 2} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x-4)}{\cancel{x-2}}$

$= \lim_{x\to 2} x - 4 = -2$

(4) $\lim_{x\to -5} \frac{2x^2 + 9x - 5}{x + 5} = \lim_{x\to -5} \frac{(2x-1)(x+5)}{x+5}$

$= \lim_{x\to -5} 2x - 1 = -11$

(5) $\lim_{x\to 1} \frac{x^3 - 1}{x - 1} = \lim_{x\to 1} \frac{(x-1)(x^2+x+1)}{x-1}$

$= \lim_{x\to 1} x^2 + x + 1$

$= 3$

(6) $\lim_{x\to 3} \frac{x^2 - 4x + 3}{x^2 - 2x - 3} = \lim_{x\to 3} \frac{(x-3)(x-1)}{(x-3)(x+1)}$

$= \lim_{x\to 3} \frac{x-1}{x+1} = \frac{2}{4} = \frac{1}{2}$

(7) $\lim_{x\to -2} \frac{x^3 + 8}{x + 2} = \lim_{x\to -2} \frac{(x+2)(x^2-2x+4)}{x+2}$

$= \lim_{x\to -2} x^2 - 2x + 4$

$= 4 + 4 + 4 = 12$

(8) $\lim_{x\to 3} \frac{x^4 - 81}{x - 3} = \lim_{x\to 3} \frac{(x-3)(x^3+3x^2+9x+27)}{x-3}$

$= \lim_{x\to 3} x^3 + 3x^2 + 9x + 27$

$= 27 + 27 + 27 + 27 = 108$

(9) $\lim_{x\to 0}((x^2 - 2)^2 + 6) = 10$

(10) $\lim_{x\to 0} \frac{5x}{x} = \lim_{x\to 0} 5 = 5$

(11) $\lim_{x\to 0} \frac{17x}{2x} = \frac{17}{2}$

(12) $\lim_{x\to 0} \frac{-317x}{422x} = \frac{-317}{422}$

(13) $\lim_{x\to 0} \frac{-317x - 3}{422x + 5} = \frac{-3}{5}$

(14) $\lim_{x\to\infty} \frac{x + 2}{x - 2} = \lim_{x\to\infty} \frac{\frac{x}{x} + \frac{2}{x}}{\frac{x}{x} - \frac{2}{x}}$

$= 1$

(15) $\lim_{x\to\infty} \frac{3x^2+2x-5}{5x^2+3x+1} = \frac{3}{5}$

(16) $\lim_{x\to\infty} \frac{x^2-7x+11}{3x^2+10} = \frac{1}{3}$

(17) $\lim_{x\to\infty} \frac{2x^3-5x+7}{7x^3+2x^2-6} = \frac{2}{7}$

(18) $\lim_{x\to\infty} \frac{(3x-1)(4x-5)}{(x+6)(x-3)} = \lim_{x\to\infty} \frac{12x^2-19x+5}{x^2+3x-18}$

$= 12$

Show the following equalities are true:

(1) $\lim_{x\to 0} \frac{\sqrt{3+x}-\sqrt{3}}{x} = \frac{1}{2\sqrt{3}}$ (hint: multiply the top and the bottom by the conjugate of the numerator. So, multiply the expression by $\frac{\sqrt{3+x}+\sqrt{3}}{\sqrt{3+x}+\sqrt{3}}$)

$$\lim_{x\to 0} \frac{\sqrt{3+x}-\sqrt{3}}{x} \cdot \frac{\sqrt{3+x}+\sqrt{3}}{\sqrt{3+x}+\sqrt{3}} = \lim_{x\to 0} \frac{\cancel{3}+x\cancel{-3}}{x(\sqrt{3+x}+\sqrt{3})} = \lim_{x\to 0} \frac{1}{\sqrt{3+x}+\sqrt{3}} = \frac{1}{2\sqrt{3}}$$

(2) $\lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} = \cancel{\frac{1}{2\sqrt{3}}}\ \frac{1}{2\sqrt{x}}$

$$\lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} \cdot \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} = \lim_{h\to 0} \frac{\cancel{x}+h\cancel{-x}}{\cancel{h}(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

(3) $\lim_{x\to\infty} \frac{x}{\sqrt{4x^2+1}-1} = \frac{1}{2}$

$$\lim_{x\to\infty} \frac{x}{\sqrt{4x^2+1}-1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{1}{\frac{\sqrt{4x^2+1}}{x} - \frac{1}{x}} = \lim_{x\to\infty} \frac{1}{\sqrt{\frac{4x^2+1}{x^2}} - \frac{1}{x}} = \lim_{x\to\infty} \frac{1}{\sqrt{4+\frac{1}{x^2}} - \frac{1}{x}} = \frac{1}{2}$$

(4) $\lim_{x\to\infty} \sqrt{n^2+1}-n = 0$ (hint: multiply by the fraction $\frac{\sqrt{n^2+1}+n}{\sqrt{n^2+1}+n}$. This is the conjugate over the conjugate)

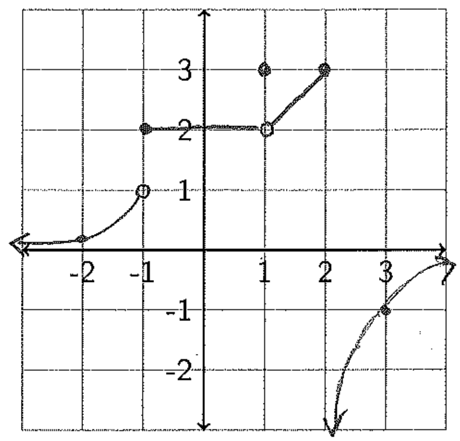
$$\lim x\to\infty(\sqrt{n^2+1}-n) \cdot \frac{\sqrt{n^2+1}+n}{\sqrt{n^2+1}+n} = \lim_{x\to\infty} \frac{n^2+1-n^2}{\sqrt{n^2+1}+n} = \lim_{x\to\infty} \frac{1}{\sqrt{n^2+1}+n} \cdot \frac{1/n}{1/n}$$

$$= \lim_{x\to\infty} \frac{1/n}{\sqrt{\frac{n^2+1}{n^2}}+1} = \lim_{x\to\infty} \frac{1/n}{\sqrt{1+\frac{1}{n^2}}+1} = 0$$

Consider the piecewise function

$$f(x) = \begin{cases} \frac{1}{x^2} & \text{if } x < -1 \\ 2 & \text{if } -1 \le x < 1 \\ 3 & \text{if } x = 1 \\ x+1 & \text{if } 1 < x \le 2 \\ \frac{-1}{(x-2)^2} & \text{if } x > 2 \end{cases}$$

First, sketch the graph of this function, then determine the following limits.



(1) $\lim_{x \to -1^-} f(x) = 1$

(2) $\lim_{x \to -1^+} f(x) = 2$

(3) $\lim_{x \to -1} f(x) = \text{DNE}$

(4) $\lim_{x \to 1^-} f(x) = 2$

(5) $\lim_{x \to 1^+} f(x) = 2$

(6) $\lim_{x \to 1} f(x) = 2$

(7) $\lim_{x \to 2^-} f(x) = 3$

(8) $\lim_{x \to 2^+} f(x) = -\infty$

(9) $\lim_{x \to 2} f(x) = \text{DNE}$

(10) $\lim_{x \to -3} f(x) = \frac{1}{(-3)^2} = 1/9$

(11) $\lim_{x \to 5} f(x) = \frac{-1}{(5-2)^2} = -\frac{1}{9}$

(12) $\lim_{x \to 1.5} f(x) = 1.5+1 = 2.5$