

Practice with Limits

Evaluate the following limits

$$(1) \lim_{x \rightarrow 1} (6x^2 - 4x + 3)$$

$$(8) \lim_{x \rightarrow 3} \frac{x^4 - 81}{x - 3}$$

$$(2) \lim_{x \rightarrow 7} \frac{x^2 - 49}{x - 7}$$

$$(9) \lim_{x \rightarrow 0} ((x^2 - 2)^2 + 6)$$

$$(3) \lim_{x \rightarrow 2} \frac{x^2 - 6x + 8}{x - 2}$$

$$(10) \lim_{x \rightarrow 0} \frac{5x}{x}$$

$$(4) \lim_{x \rightarrow -5} \frac{2x^2 + 9x - 5}{x + 5}$$

$$(11) \lim_{x \rightarrow 0} \frac{17x}{2x}$$

$$(5) \lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1}$$

$$(12) \lim_{x \rightarrow 0} \frac{-317x}{422x}$$

$$(6) \lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{x^2 - 2x - 3}$$

$$(13) \lim_{x \rightarrow 0} \frac{-317x - 3}{422x + 5}$$

$$(7) \lim_{x \rightarrow -2} \frac{x^3 + 8}{x + 2}$$

$$(14) \lim_{x \rightarrow \infty} \frac{x + 2}{x - 2}$$

$$(15) \lim_{x \rightarrow \infty} \frac{3x^2 + 2x - 5}{5x^2 + 3x + 1}$$

$$(17) \lim_{x \rightarrow \infty} \frac{2x^3 - 5x + 7}{7x^3 + 2x^2 - 6}$$

$$(16) \lim_{x \rightarrow \infty} \frac{x^2 - 7x + 11}{3x^2 + 10}$$

$$(18) \lim_{x \rightarrow \infty} \frac{(3x - 1)(4x - 5)}{(x + 6)(x - 3)}$$

Show the following equalities are true:

$$(1) \lim_{x \rightarrow 0} \frac{\sqrt{3+x} - \sqrt{3}}{x} = \frac{1}{2\sqrt{3}} \text{ (hint: multiply the top and the bottom by the conjugate of the numerator. So, multiply the expression by } \frac{\sqrt{3+x} + \sqrt{3}}{\sqrt{3+x} + \sqrt{3}} \text{)}$$

$$(2) \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \frac{1}{2\sqrt{x}}$$

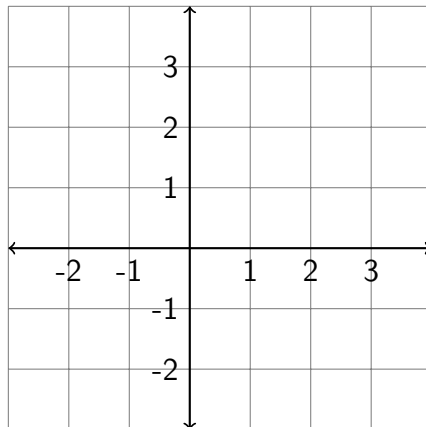
$$(3) \lim_{x \rightarrow \infty} \frac{x}{\sqrt{4x^2 + 1} - 1} = \frac{1}{2}$$

$$(4) \lim_{x \rightarrow \infty} \sqrt{n^2 + 1} - n = 0 \text{ (hint: multiply by the fraction } \frac{\sqrt{n^2+1}+n}{\sqrt{n^2+1}+n} \text{. This is the conjugate over the conjugate)}$$

Consider the piecewise function

$$f(x) = \begin{cases} \frac{1}{x^2} & \text{if } x < -1 \\ 2 & \text{if } -1 \leq x < 1 \\ 3 & \text{if } x = 1 \\ x + 1 & \text{if } 1 < x \leq 2 \\ \frac{-1}{(x-2)^2} & \text{if } x > 2 \end{cases}$$

First, sketch the graph of this function, then determine the following limits.



- (1) $\lim_{x \rightarrow -1^-} f(x) =$
- (2) $\lim_{x \rightarrow -1^+} f(x) =$
- (3) $\lim_{x \rightarrow -1} f(x) =$
- (4) $\lim_{x \rightarrow 1^-} f(x) =$
- (5) $\lim_{x \rightarrow 1^+} f(x) =$
- (6) $\lim_{x \rightarrow 1} f(x) =$
- (7) $\lim_{x \rightarrow 2^-} f(x) =$
- (8) $\lim_{x \rightarrow 2^+} f(x) =$
- (9) $\lim_{x \rightarrow 2} f(x) =$
- (10) $\lim_{x \rightarrow -3} f(x) =$
- (11) $\lim_{x \rightarrow 5} f(x) =$
- (12) $\lim_{x \rightarrow 1.5} f(x) =$