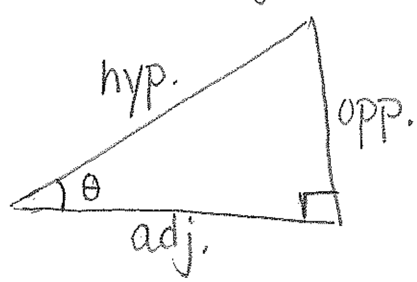
Jan 8, 2014

Trigonometry = triangle measure

All starts w/ right triangle



* trig is based on the fact that different sized triangles w/ the same angle measurements are similar (the ratios btwn their sides are the same)

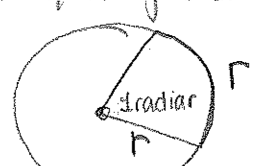
$\sin\theta = \frac{opp}{hyp}$ $\cos\theta = \frac{adj}{hyp}$ $\tan\theta = \frac{opp}{adj}$ (SOH CAH TOA)

$\csc\theta = \frac{hyp}{opp}$ $\sec\theta = \frac{hyp}{adj}$ $\cot\theta = \frac{adj}{opp}$

So, given one angle, we can figure out every ratio in a right triangle.

- Remember adj, opp are relative to whatever angle you pick.
- in calculus, we like to think about radians instead of degrees

Fun Fact: the angle measure 1 radian is the size of angle s.t. sector of circle w/ angle 1 radian intersects arc of length r

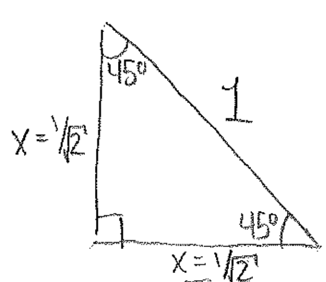


$\Rightarrow 2\pi$ radians $= 360°$

So 1 radian $= \frac{180}{\pi}$ degrees

$1° = \frac{\pi}{180}$ radians

Common values sin, cos, tan



$45° = \frac{\pi}{4}$ radians

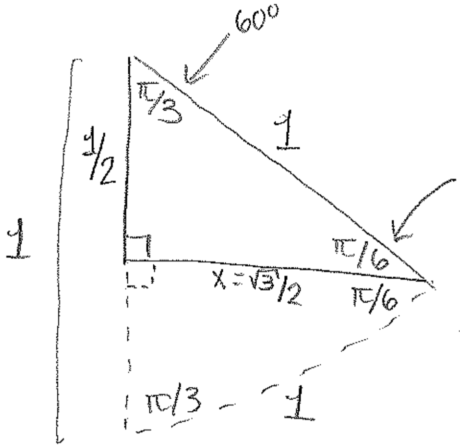
pythagorean thm: $a^2+b^2=c^2$

$x^2+x^2=1^2 \Leftrightarrow 2x^2=1 \Leftrightarrow x=\pm\frac{1}{\sqrt{2}}$ (only pos. makes sense)

$\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\tan\left(\frac{\pi}{4}\right) = 1$



$\left(\frac{1}{2}\right)^2 + x^2 = 1$

$x^2 = \frac{3}{4}$

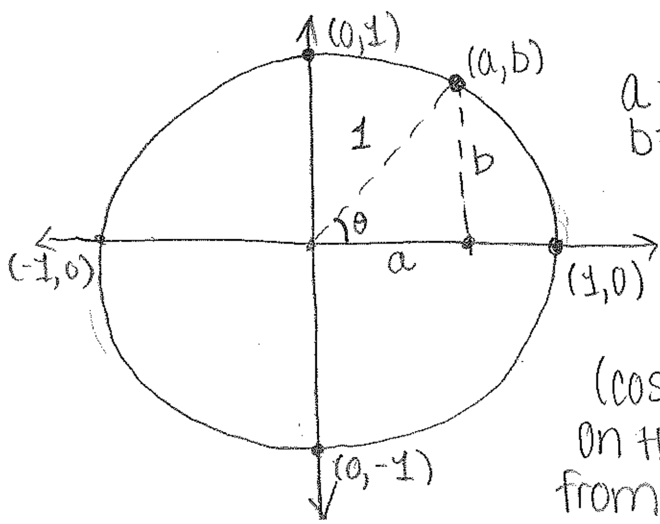
$x = \pm\frac{\sqrt{3}}{2}$

$\sin(\pi/6) = 1/2$ $\qquad$ $\sin(\pi/3) = \sqrt{3}/2$

$\cos(\pi/6) = \sqrt{3}/2$ $\qquad$ $\cos(\pi/3) = 1/2$

$\tan(\pi/6) = 1/\sqrt{3}$ $\qquad$ $\tan(\pi/3) = \sqrt{3}$

The Unit Circle (centered at (0,0) with radius 1)



$a = \cos\theta$
$b = \sin\theta$

$(\cos\theta, \sin\theta)$ is the coordinate on the unit circle θ radians from the pos. x-axis.

Fact: $\cos(\pi - \theta) = -\cos\theta$ $\qquad$ $\sin(\pi - \theta) = \sin\theta$

$\cos(-\theta) = \cos\theta$ $\qquad$ $\sin(-\theta) = -\sin\theta$

$\cos(2\pi \underbrace{n}_{\text{integer}} + \theta) = \cos\theta$ $\qquad$ $\sin(2\pi \underbrace{n}_{\text{integer}} + \theta) = \sin\theta$

<u>VERY IMPORTANT</u>: unit circle equation: $x^2 + y^2 = 1$

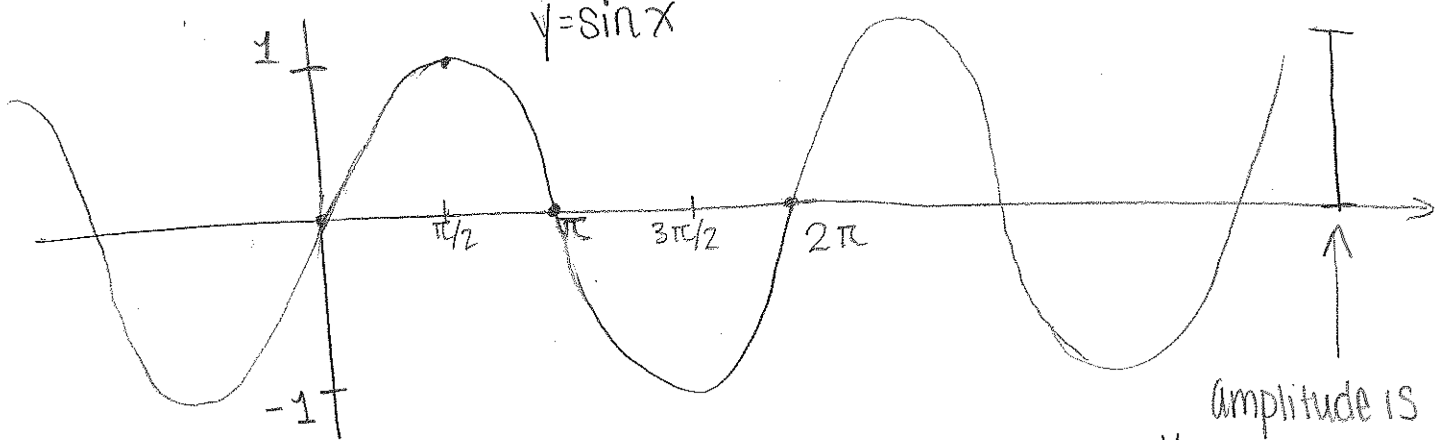
<u>so</u>....

$\cos^2\theta + \sin^2\theta = 1$

- Unit Circle Worksheet

Graphs of sin, cos, tan

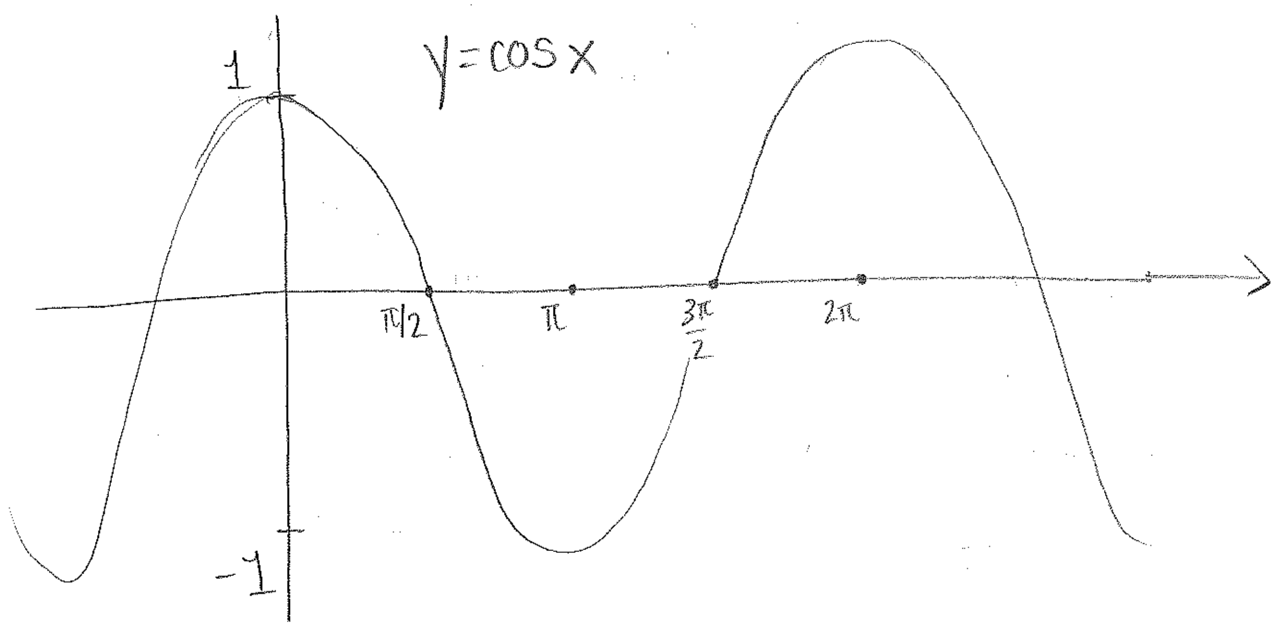
$y = \sin x$



period: length on x-axis of one cycle (one repetition of pattern) 2π

amplitude is ½ the distance from highest pt to lowest pt
1

$y = \cos x$



$y = a \cdot \sin(b \cdot \theta + c) + d$

amplitude $= a$

period $= \frac{2\pi}{b}$

$$y = \underbrace{2}_{(3)}\sin\left(\underbrace{\tfrac{1}{2}}_{(1)}\theta + \underbrace{\tfrac{\pi}{6}}_{(2)}\right)$$

(2) shifts graph left by $\pi/6$

(1) stretches out graph by a factor of 2

(3) expands vertically by a factor of two.

Period: 4π

Amplitude: 2

Identities to know

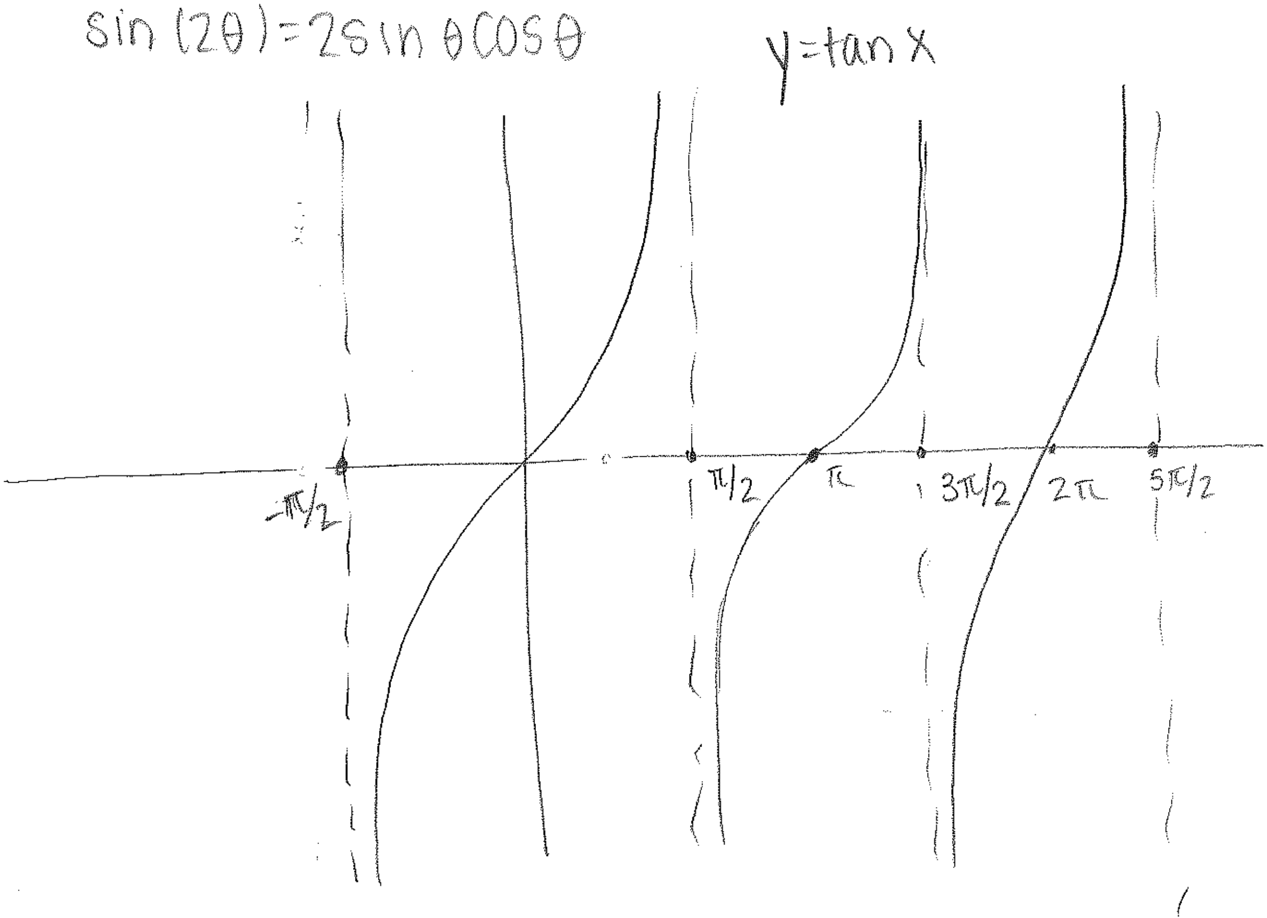
$\cos(-\theta) = \cos(\theta)$ (even)

$\sin(-\theta) = -\sin(\theta)$ (odd)

$\cos^2\theta + \sin^2\theta = 1$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$\sin(2\theta) = 2\sin\theta\cos\theta$

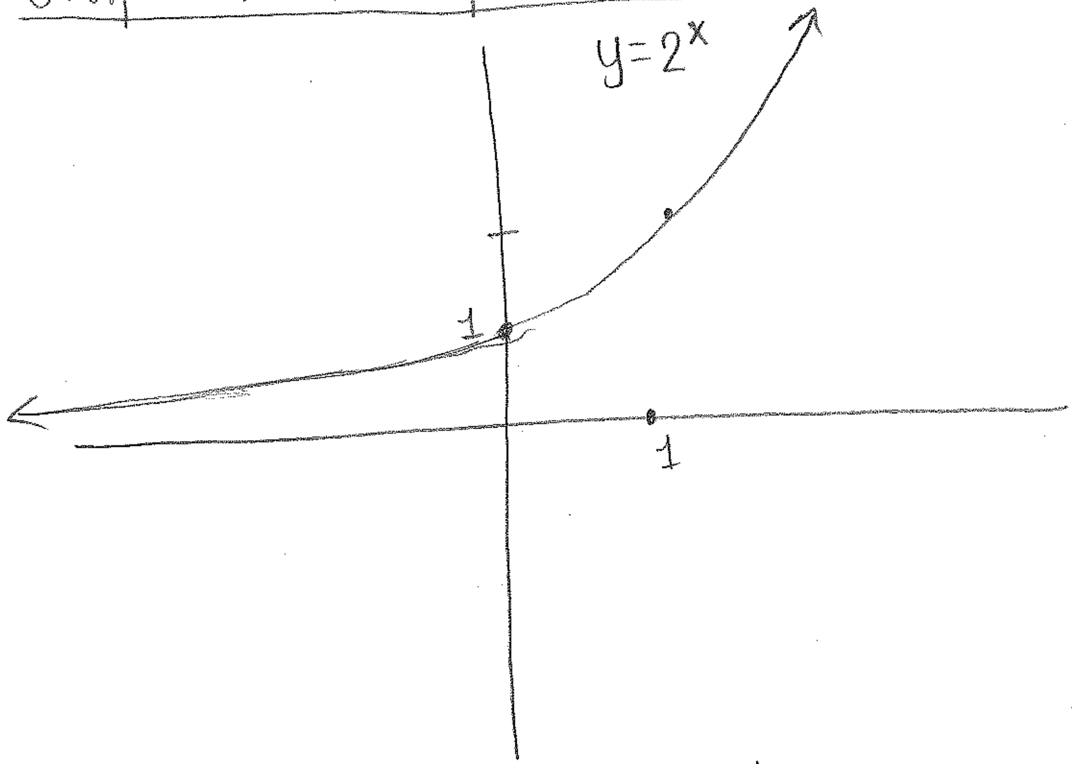


Exponents

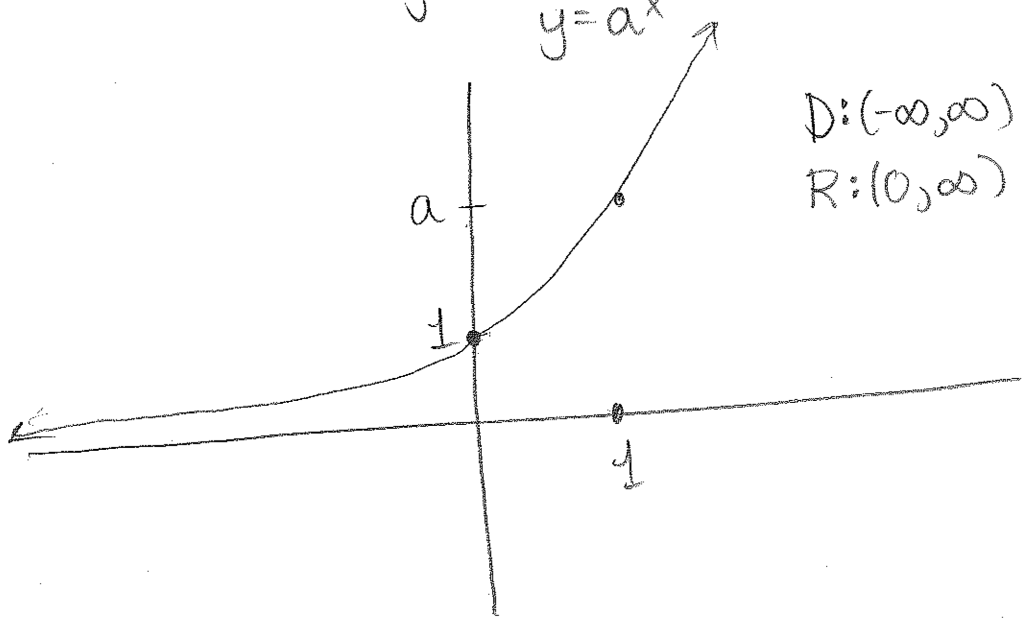
$$\underbrace{a^n}_{\text{real \#}} = \underbrace{a \cdot a \cdot \ldots \cdot a}_{n}$$ (n ← pos. integer) (WeBWork a^n or a**n)

Identities	Example
$a^n \cdot a^m = a^{n+m}$	$2^3 2^5 = (2\cdot2\cdot2)\cdot(2\cdot2\cdot2\cdot2\cdot2)$ $= 2^8$
$(a^n)^m = a^{n\cdot m}$	$(3^2)^3 = (3^2)\cdot(3^2)\cdot(3^2)$ $= (3\cdot3)(3\cdot3)(3\cdot3)$
$a^n \cdot b^n = (ab)^n$	$5^3 \cdot 2^3 = 5\cdot5\cdot5\cdot2\cdot2\cdot2$ $= (5\cdot2)(5\cdot2)(5\cdot2)$ $= (5\cdot2)^3$
$a^0 = 1$	$a^n = a^{n+0} = a^n \cdot a^0$ so $a^0 = 1$
$a^{-n} = \frac{1}{a^n}$	$a^n \cdot a^{-n} = a^{n-n} = a^0 = 1$ $a^{-n} = 1/a^n$
$a^{1/n} = \sqrt[n]{a}$	$(a^n)^{1/n} = a^1 = a$
$a^{m/n} = \sqrt[n]{a^m}$ $= \sqrt[m]{a^n}$	$2^{3/2} = \sqrt{2^3}$ OR $\sqrt{2}^3$ $= \sqrt{8} = 2\sqrt{2}$

Graph of an exponential function



in general... $1<a$



D: $(-\infty,\infty)$
R: $(0,\infty)$

an important #: $e = 2.71828183\ldots$

$f(x)=e^x$ will be our fav exponential function.

* different story for $0<a<1$, $a<0$, $a=0$, $a=1$