out of 33

HW 2

(1) $\frac{dy}{dx} = \frac{1}{2}(x^2-1)$ $\quad y(0)=2$

$2\,dy = (x^2-1)\,dx$ Separate variables: 2 pt

$\int 2\,dy = \int (x^2-1)\,dx$

$2y + C_1 = \frac{x^3}{3} - x + C_2$ antiderivatives: 1 pt each
1 pt for +C

$y = \frac{\frac{x^3}{3} - x + C}{2}$

$y = \frac{x^3}{6} - \frac{x}{2} + C$ solve for y: 1 pt

$y(0)=2$ so $2 = \frac{0^3}{6} - \frac{0}{2} + C$ find C: 2 pt

$2 = C$

$\boxed{y = \frac{x^3}{6} - \frac{x}{2} + 2}$ answer: 1 pt

out of 9

2. $y^2(y^2-4) = x^2(x^2-5)$

$\frac{d}{dx}(y^2(y^2-4)) = \frac{d}{dx}(x^2(x^2-5))$ derivative:

$\frac{d}{dx}(y^4-4y^2) = \frac{d}{dx}(x^4-5x^2)$

Chain rule: 2pt
Everything else: 2pt

$4y^3 \cdot \frac{dy}{dx} - 8y\frac{dy}{dx} = 4x^3 - 10x$

$\frac{dy}{dx}(4y^3-8y) = 4x^3-10x$

$\frac{dy}{dx} = \frac{4x^3-10x}{4y^3-8y}$ Solve for $\frac{dy}{dx}$: 2 pt

Plug in $(0,-2)$

$\left.\frac{dy}{dx}\right|_{(0,-2)} = \frac{4(0)^3-10(0)}{4(-2)^3-8(-2)}$ Slope: 1 pt

$= 0$

$y+2 = 0(x-0)$

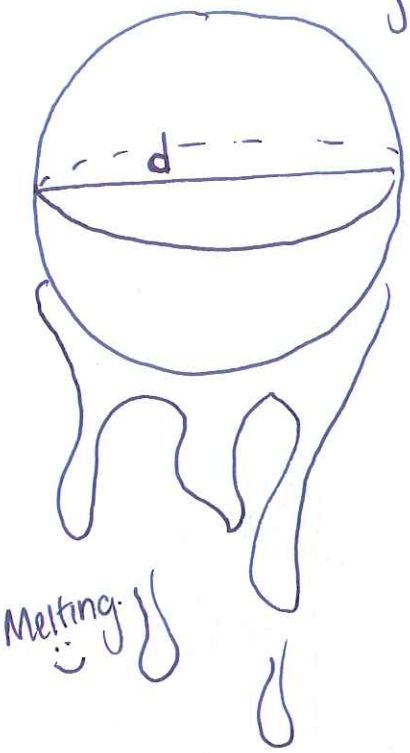
$\boxed{y=-2}$

tangent line

equation: 1 pt

out of 8

(3)



S = surface area

$S = 4\pi r^2$ (r: radius) $\quad d = 2r$ so $\frac{d}{2} = r$

$$\boxed{S = 4\pi\left(\frac{d}{2}\right)^2}$$ relating equation: 2 pt

$$S = \pi d^2$$

derivative w.r.t. t: 1 pt

$$\frac{d}{dt}S = \frac{d}{dt}\pi d^2$$ not the best choice of letter. oh well.

$$\frac{dS}{dt} = 2\pi d \cdot \frac{dd}{dt}$$

derivative: 2 pt

Now, want $\frac{dd}{dt}$ when $d = 10$, $\frac{dS}{dt} = 1$

$(1) = 2\pi(10)\cdot\frac{dd}{dt}$ plug-in values and solve for $\frac{dd}{dt}$: 2 pt

$$\boxed{\frac{1}{20\pi} = \frac{dd}{dt}}$$

$$\boxed{\frac{1}{20\pi} \text{ cm/min}}$$

out of 7

(4) $\frac{dy}{dx} = 2x - 3y$

start at $(4, 3)$

$\Delta x = 1/2$

want $y(6)$

x	y	$\frac{dy}{dx}$	Δy
4	3	$2(4) - 3(3) = -1$	$(-1)(1/2) = -1/2$
4.5	2.5	$2(4.5) - 3(2.5) = 1.5$	$(1.5)(1/2) = .75$
5	3.25	$2(5) - 3(3.25) = .25$	$(.25)(1/2) = .125$
5.5	3.375	$2(5.5) - 3(3.375) = 0.875$	$(.875)(1/2) = .4375$
6	3.8125		

So $y(6) \approx 3.8125$

1/2 pt for each correct entry (if one entry is wrong but rest does the process correctly, mark correct)

9 pt.