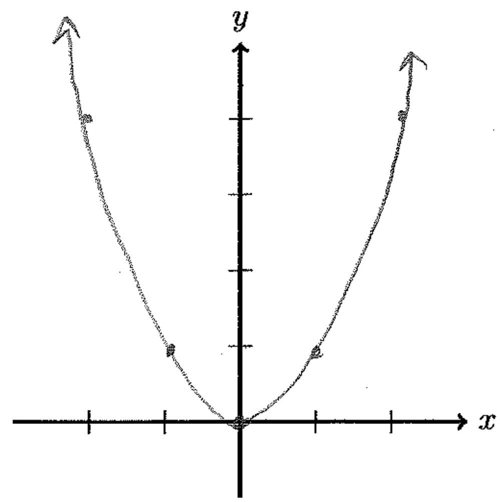
Math 3 - Day 1 - WARMUP

Calculate the given functions at the given x-values, and then plot the corresponding points.

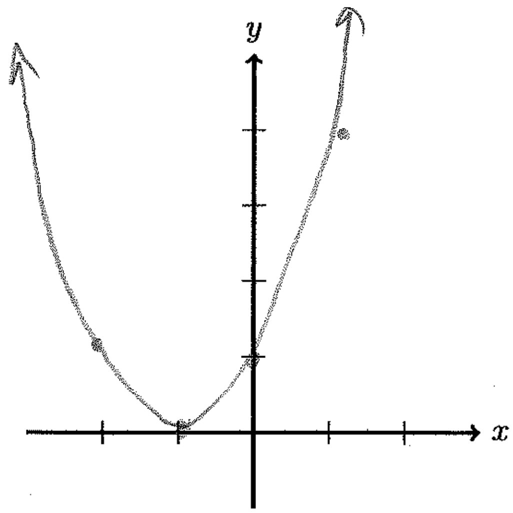
$f(x) = x^2$

x	$f(x)$
-2	4
-1	1
0	0
1	1
2	4



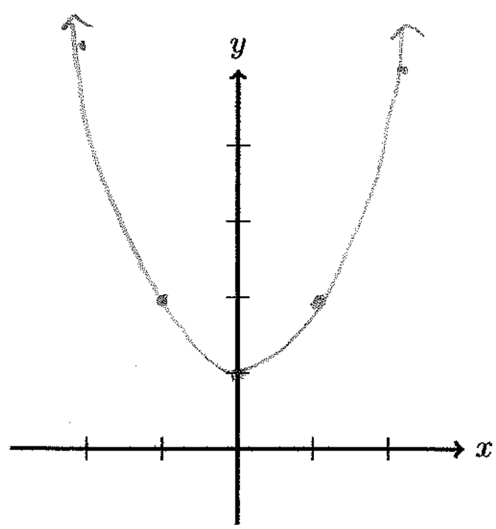
$f(x) = (x+1)^2$

x	$f(x)$
-2	1
-1	0
0	1
1	4

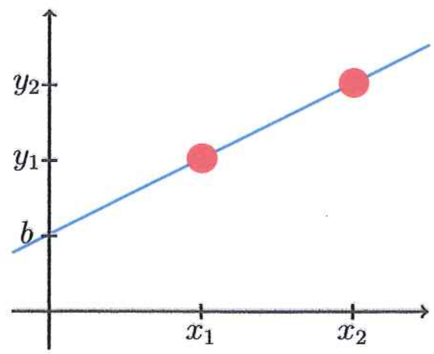


$f(x) = x^2 + 1$

x	$f(x)$
-2	5
-1	2
0	1
1	2
2	5



The Simplest Functions: Lines!



Recall: Two points define a line!

Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ (rise/run)

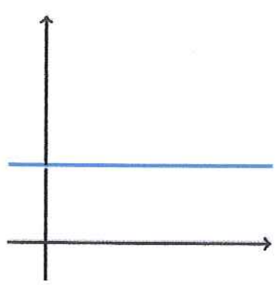
Point-slope Form: $y - y_1 = m(x - x_1)$ (good for writing down lines)

Slope-intercept Form: $y = mx + b$ (good for graphing)

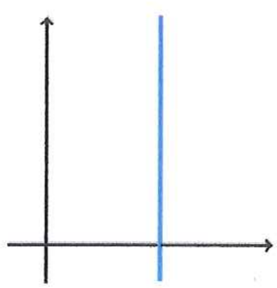
General Form: $Ax + By + C = 0$ (good when the slope is ∞)

Special Cases of Lines

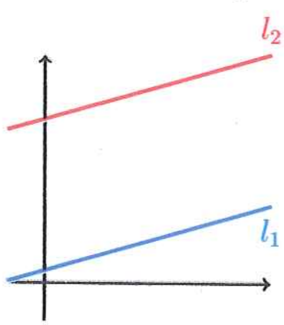
Constant Functions: $m = 0$



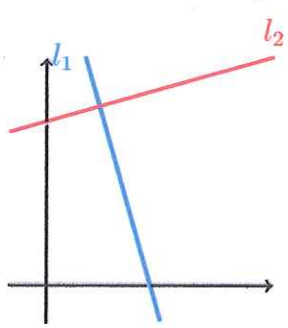
Vertical Lines: $m = \infty$



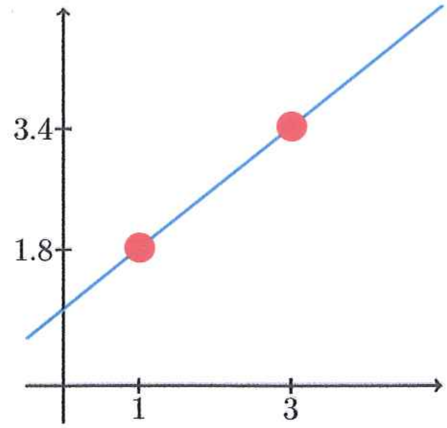
Parallel Lines: $m_1 = m_2$



Perpendicular Lines: $m_1 = -1/m_2$



Practice with Lines



(1) Find the equation of the above line using the different forms:

Slope: $m = \frac{3.4-1.8}{3-1} = \frac{1.6}{2} = .8$

Point-slope Form: $y - 1.8 = (.8)(x-1)$ OR $y - 3.4 = (.8)(x-3)$

Slope-intercept Form: $y = (.8)x + 1$

General Form: $y - (.8)x - 1 = 0$

(2) Find the equation of the line that is perpendicular to $y = \frac{3}{2}x + 1$ that goes through the point $(1, 1)$.

$m = -\frac{2}{3}$

$y - 1 = (-\frac{2}{3})(x-1)$ (Point-Slope)

$y = (-\frac{2}{3})x + \frac{5}{3}$ (slope-int)

Knowing Graphs of Functions

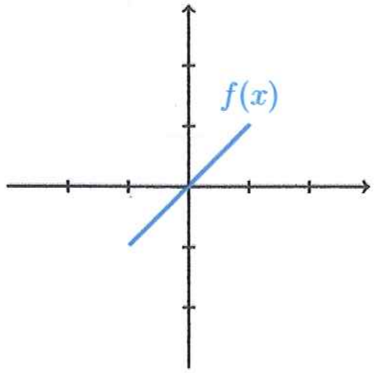
For each of the functions below, list the domain and range, and sketch it's corresponding graph.

Polynomials: $f(x) = x^2$, $f(x) = x^3$, $f(x) = x^4$, $f(x) = x^5$
Rationals: $f(x) = \frac{1}{x}$, $f(x) = \frac{1}{x^2}$
Roots: $f(x) = x^{1/2} = \sqrt{x}$, $f(x) = x^{1/3} = \sqrt[3]{x}$
Trigonometry: $f(x) = \sin x$, $f(x) = \cos x$, $f(x) = \tan x$
Logarithms, Exponential: $f(x) = \ln x$, $f(x) = e^x$

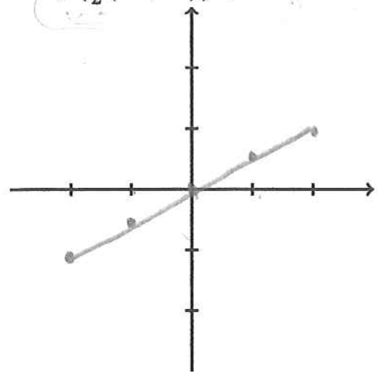
New Functions from Old

Ex: Transform the graph of $f(x)$ into the graph of $-f(\frac{1}{2}(x+1)) + 2$.

D: [-1,1]
R: [-1,1]



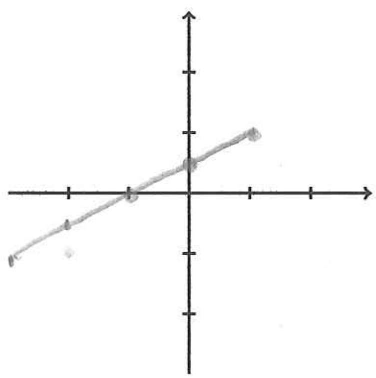
$\xrightarrow{f(\frac{1}{2}x)}$



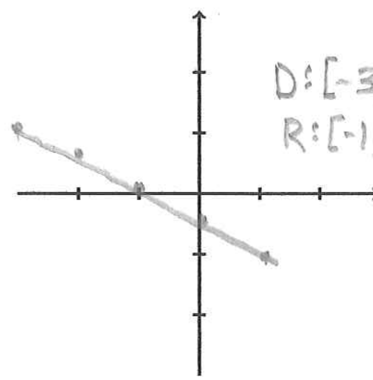
D: [-2,2]
R: [-1,1]

D: [-3,1]
R: [-1,1]

$\xrightarrow{f(\frac{1}{2}(x+1))}$



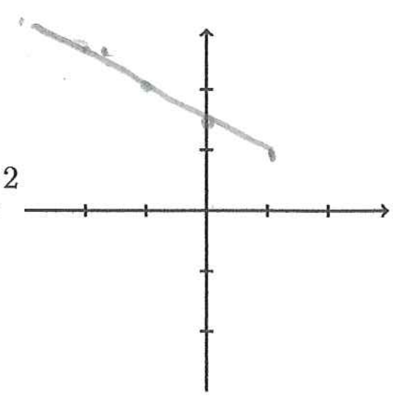
$\xrightarrow{-f(\frac{1}{2}(x+1))}$



D: [-3,1]
R: [-1,1]

D: [-3,1]
R: [1,3]

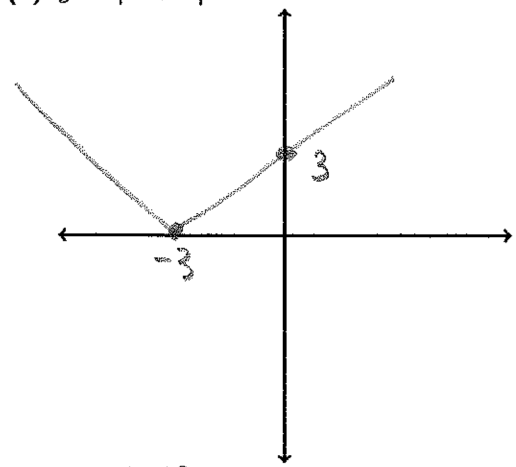
$\xrightarrow{-f(\frac{1}{2}(x+1))+2}$



Practice

(1) Sketch the graph the following functions. Find the domain and range of each.

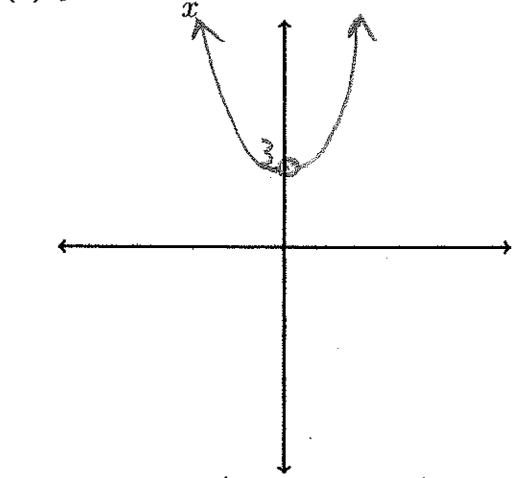
(a) $y = |x+3|$



Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

(b) $y = \dfrac{(2x)^3 + 3x}{x} = 8x^2 + 3$



$x \neq 0$

Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $(3, \infty)$

(2) Let $f(x) = \dfrac{x+1}{3x-2}$ and $g(x) = \dfrac{1}{x}$.

(a) Calculate $(f \circ g)(x)$ and $(g \circ f)(x)$.

$f \circ g(x) = \dfrac{\left(\frac{1}{x}\right)+1}{\frac{3}{x}-2} = \dfrac{1+x}{3-2x}$

$(g \circ f)(x) = \dfrac{1}{\left(\frac{x+1}{3x-2}\right)} = \dfrac{3x-2}{x+1}$

(b) What is the domain of $(g \circ f)(x)$? (Hint: Be careful! The domain of $(g \circ f)(x)$ will be those x's where $f(x)$ exists AND $(g \circ f)(x)$ exists.

$\dfrac{x+1}{3x-2} \neq 0$

$x \neq -1$

$3x - 2 \neq 0$

$x \neq \frac{2}{3}$

$D: (-\infty, -1) \cup (-1, \frac{2}{3}) \cup (\frac{2}{3}, \infty)$

(3) Let $f(x) = \dfrac{x+1}{3x-2}$.

(a) Calculate $f^{-1}(x)$.

$x = \dfrac{y+1}{3y-2} \Rightarrow 3xy - 2x = y+1$

$-1 - 2x = y - 3xy$

$-1 - 2x = y(1-3x)$

$y = \dfrac{-1-2x}{1-3x} = f^{-1}(x)$

(b) Check your answer to (a) by explicit calculating both $f \circ f^{-1}$ and $f^{-1} \circ f$. (You should get x both times).

(c) If $(f \circ g)(x) = x + 2$, what is $g(x)$? (Hint: Since $(f \circ g)(x)$, we know that $g(x) = f^{-1}(f(g(x))) = f^{-1}(x+2)$

$\dfrac{-1-2(x+2)}{1-3(x+2)} = g(x)$

(4) (a) Does $f(x) = x^2$ have an inverse? If yes, what is it? If no, why not?

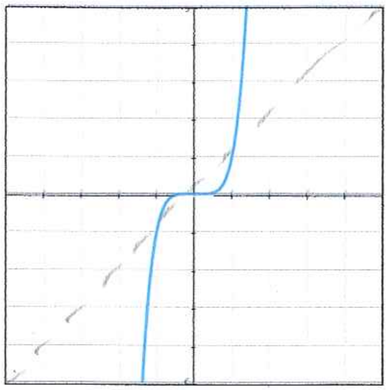
No, x^2 is not 1-1

(b) Consider $f(x) = x^2$ with Domain=[0,∞). Does this function have an inverse? If yes, what is it? If no, why not?

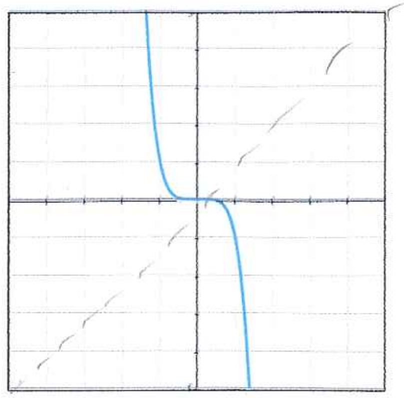
Yes, $\sqrt{x}$

(5) Match each graph to it's inverse.

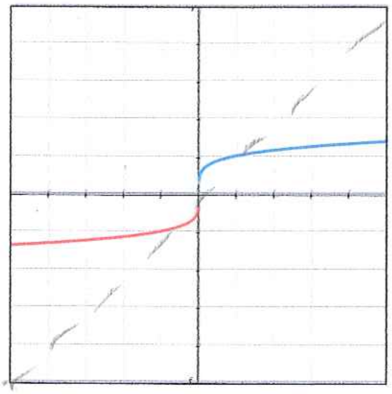
A.



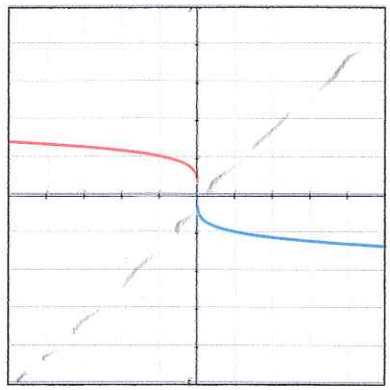
B.



C.



D.



A inverse of C

B inverse of D