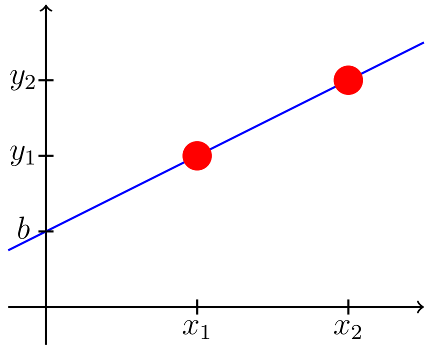
The Simplest Functions: Lines!



Recall: Two points define a line!

Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ (rise/run)

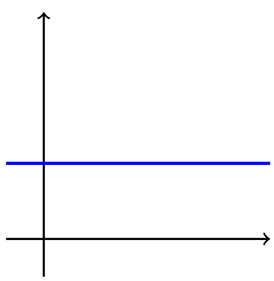
Point-slope Form: $y - y_1 = m(x - x_1)$ (good for writing down lines)

Slope-intercept Form: $y = mx + b$ (good for graphing)

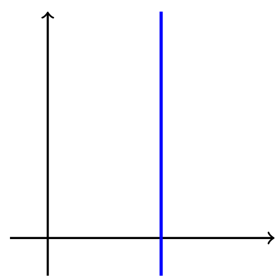
General Form: $Ax + By + C = 0$ (good when the slope is ∞)

Special Cases of Lines

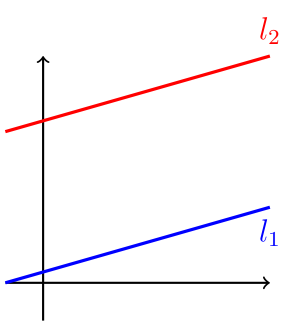
Constant Functions: $m = 0$



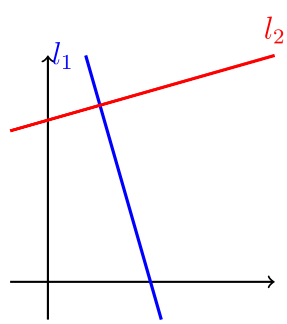
Vertical Lines: $m = \infty$



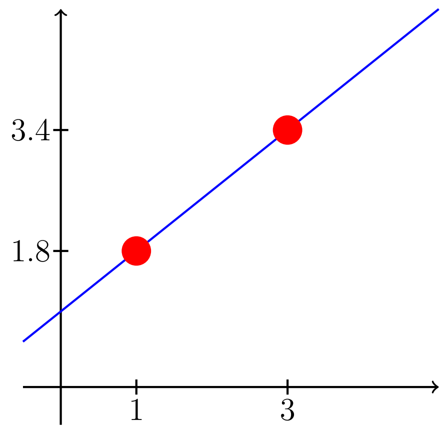
Parallel Lines: $m_1 = m_2$



Perpendicular Lines: $m_1 = -1/m_2$



Practice with Lines



(1) Find the equation of the above line using the different forms:

Slope:

Point-slope Form:

Slope-intercept Form:

General Form:

(2) Find the equation of the line that is perpendicular to $y = \frac{3}{2}x + 1$ that goes through the point $(1, 1)$.

Knowing Graphs of Functions

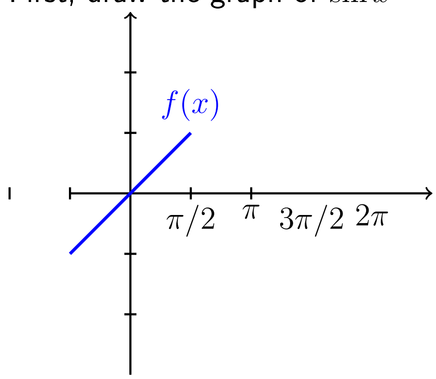
For each of the functions below, ensure you can sketch the graph and list the function's domain and range.

Polynomials: $f(x) = x^2$, $f(x) = x^3$, $f(x) = x^4$, $f(x) = x^5$
Rationals: $f(x) = \frac{1}{x}$, $f(x) = \frac{1}{x^2}$
Roots: $f(x) = x^{1/2} = \sqrt{x}$, $f(x) = x^{1/3} = \sqrt[3]{x}$
Trigonometry: $f(x) = \sin x$, $f(x) = \cos x$, $f(x) = \tan x$
Logarithms, Exponential: $f(x) = \ln x$, $f(x) = e^x$

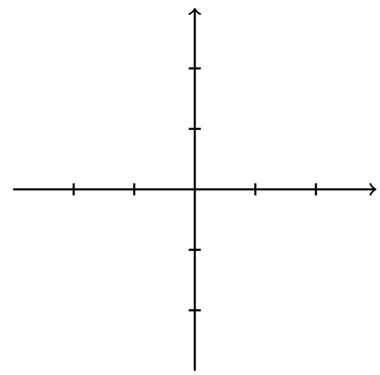
New Functions from Old

Ex: Transform the graph of $f(x)$ into the graph of $-f(\frac{1}{2}(x+1)) + 2$.

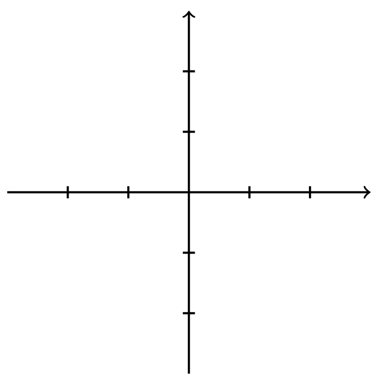
First, draw the graph of $\sin x$



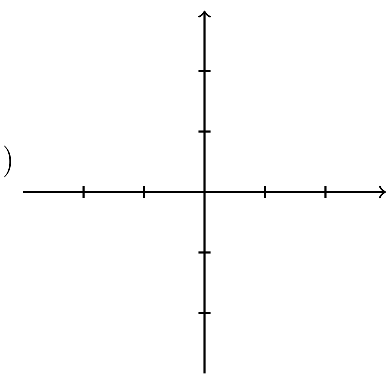
(1) $f(\frac{1}{2}x)$



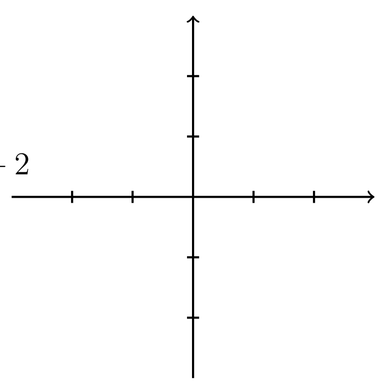
(2) $f(\frac{1}{2}(x+1))$



(3) $-f(\frac{1}{2}(x+1))$



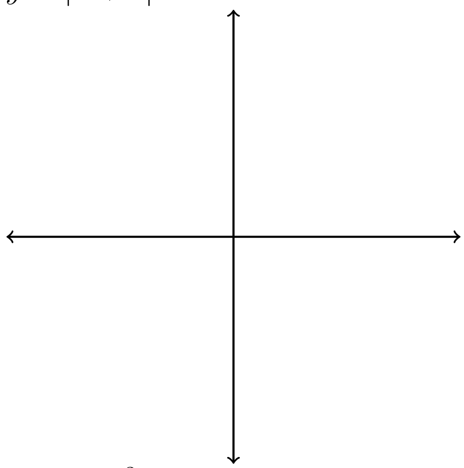
(4) $-f(\frac{1}{2}(x+1)) + 2$



Practice

(1) Sketch the graph the following functions. Find the domain and range of each.

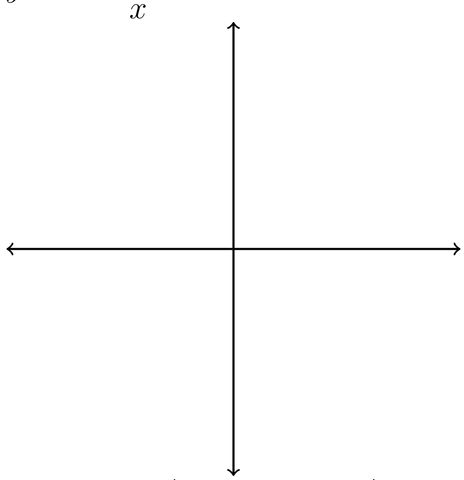
(a) $y = |x + 3|$



Domain:

Range:

(b) $y = \dfrac{(2x)^3 + 3x}{x}$



Domain:

Range:

(2) Let $f(x) = \dfrac{x+1}{3x-2}$ and $g(x) = \dfrac{1}{x}$.

(a) Calculate $(f \circ g)(x)$ and $(g \circ f)(x)$.

(b) What is the domain of $(g \circ f)(x)$? (Hint: Be careful! The domain of $(g \circ f)(x)$ will be those x's where $f(x)$ exists AND $(g \circ f)(x)$ exists.

(3) Let $f(x) = \dfrac{x+1}{3x-2}$.

(a) Calculate $f^{-1}(x)$.

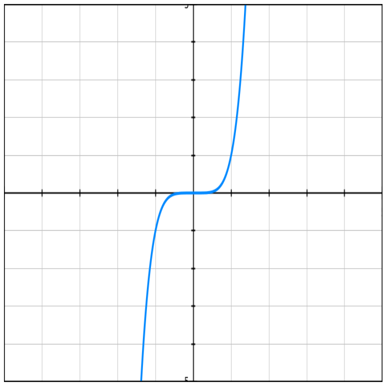
(b) Check your answer to (a) by explicit calculating both $f \circ f^{-1}$ and $f^{-1} \circ f$. (You should get x both times).

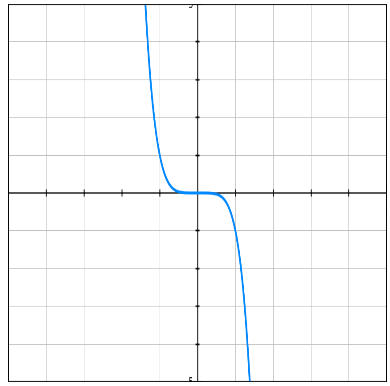
(c) If $(f \circ g)(x) = x + 2$, what is $g(x)$? (Hint: Since $(f \circ g)(x)$, we know that $g(x) = f^{-1}(f(g(x))) = f^{-1}(x+2)$

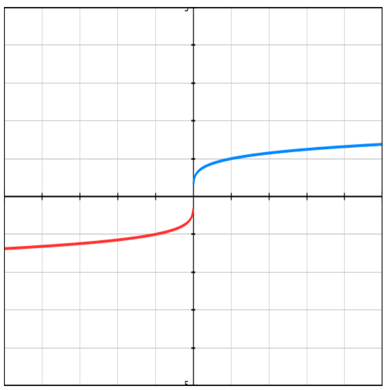
(4) (a) Does $f(x) = x^2$ have an inverse? If yes, what is it? If no, why not?

(b) Consider $f(x) = x^2$ with Domain=[0,∞). Does this function have an inverse? If yes, what is it? If no, why not?

(5) Match each graph to its inverse.

A. 

B. 

C. 

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