Final Exam

Math 3, Fall 2011

December 3, 2011

Name (Print): ____

Last First

On this the Math 3 final exam in Fall 2011, I will work individually, neither giving nor receiving help, guided by the Dartmouth Academic Honor Principle.

Signature:___________________________________

Instructor (circle):

Lahr (Sec. 1, 8:45) Crytser (Sec. 2, 11:15) Daugherty (Sec. 3, 12:30)

Instructions: You are not allowed to use calculators, books, or notes of any kind. All of your answers must be marked on the Scantron form provided. Take a moment now to print your name and section clearly on your Scantron form and on the cover of your exam booklet and sign the affirmation. You may write on the exam, but you will only receive credit on Scantron (multiple-choice) problems for what you write on the Scantron form. At the end of the exam, you must turn in both your Scantron form and your exam booklet. There are 25 multiple choice problems worth 6 each, for a total of 150. Check to see that you have 14 pages of questions plus the cover page and a blank page at the end for scrap work.

1. Let $f(x) = \dfrac{2x+1}{3x+2}$. Which of the following is the limit definition of $f'(2)$?

   (a) $\displaystyle\lim_{x\to 2} \frac{2x+1}{3x+2}$

   (b) $\displaystyle\lim_{h\to 0} \frac{1}{h}\left(\frac{2h+1}{3h+2} - \frac{2\cdot 2+1}{3\cdot 2+2}\right)$

   (c) $\displaystyle\lim_{h\to 0} \frac{1}{h}\left(\frac{2x+2h+1}{3x+3h+2} - \frac{2x+1}{3x+2}\right)$

   (d) $\displaystyle\lim_{h\to 0} \frac{1}{h}\left(\frac{2(2+h)+1}{3(2+h)+2} - \frac{2\cdot 2+1}{3\cdot 2+2}\right)$

   (e) None of the above.

2. Which of the following describes the behavior of $f(x) = \dfrac{x^2-1}{x-1}$ at the point $x = 1$?

   (a) The function f has a jump discontinuity at $x = 1$.

   (b) The function f has a continuous extension at $x = 1$.

   (c) The function f has a removable discontinuity at $x = 1$.

   (d) The function f is continuous on $(-\infty, \infty)$.

   (e) None of the above.

3. Let $f(x) = x^2 + e^x + \sin(x)$. Which of the following is $f'(x)$?

(a) $f'(x) = 2x + e^x + \cos(x)$

(b) $f'(x) = x^3/3 + e^x - \cos(x)$

(c) $f'(x) = x^3/3 + e^x - \cos(x) + C$

(d) $f'(x) = 2x + e^x + \cos(x) + C$

(e) None of the above.

4. Let $f(x) = \ln\left(\frac{x^2+3}{\sin(x)}\right)$. Which of the following is $f'(x)$?

(a) $f'(x) = \dfrac{\sin(x)}{x^2+3}$

(b) $f'(x) = \dfrac{\sin(x)}{x^2+3}\left(\dfrac{\cos(x)}{2x}\right)$

(c) $f'(x) = \dfrac{2x\sin(x) - (x^2+3)\cos(x)}{(x^2+3)\sin(x)}$

(d) $f'(x) = 2x\sin(x)\ln\left(\dfrac{x^2+3}{\sin(x)}\right)$

(e) None of the above

5. Let

$$f(x) = \begin{cases} |x| & \text{when } x \neq 0, \\ 1 & \text{when } x = 0. \end{cases}$$

Which of the following is true?

(a) The function f is not continuous at $x = 0$, because $\lim_{x \to 0} f(x)$ does not exist.

(b) The function f is not continuous at $x = 0$, because the function f is not defined at $x = 0$.

(c) The function f is not continuous at $x = 0$, because $f(0) \neq \lim_{x \to 0} f(x)$.

(d) The function f is not continuous at $x = 0$, because $\lim_{h \to 0} \dfrac{f(0+h) - f(0)}{h}$ does not exist.

(e) None of the above.

6. Which of the following is the equation of the tangent line to $f(x) = \sec(x) + 2$ through the point $(\pi/4, 2 + \sqrt{2})$?

(a) $y = (\tan(x))x + (2 + \sqrt{2})$.

(b) $y = x + 2 + \sqrt{2}$

(c) $y = \sec(x)\tan(x)(x - \pi/4) + 2 + \sqrt{2}$

(d) $y = \sqrt{2}(x - \pi/4) + 2 + \sqrt{2}$

(e) None of the above.

7. Let $f(x) = x^2 + x + 1$. Find $c \in (0, 2)$ so that

$$f'(c) = \frac{f(2) - f(0)}{2}.$$

(a) $c = 0$

(b) $c = 3/4$

(c) $c = 1$

(d) $c = 5/4$

(e) None of the above.

8. What is the maximum value of the function $f(x) = x^3 - 3x + 2$ on the interval $[1/2, 4]$?

(a) 1

(b) 0

(c) 54

(d) 5/8

(e) None of the above.

9. Suppose that polonium decays at a rate proportional to the amount present. If a sample of polonium-210 decays so that there is 50 g left after 140 days and 25 g left after 280 days, how much polonium did the sample have to begin with?

   (a) $50(e^{140/280})$ g
   (b) $100e^{0}$ g
   (c) $100e^{210/280}$ g
   (d) $(50/25)e^{140/280}$ g
   (e) None of the above

10. Solve

$$\frac{dy}{dx} = 3x^2y^2.$$

   (a) $y = 3(6x)y^2 + 3(x^2)\left(\frac{dy}{dx}\right)$
   (b) $y = \frac{1}{C - x^3}$
   (c) $y = \ln\left(\sqrt[3]{x^3 + C}\right)$
   (d) $y = \frac{1}{x} + C$
   (e) None of the above

11. Suppose that

$$\frac{dy}{dx} = x + 3y; y(0) = 1$$

Use Euler's method to approximate $y(1)$ with step size $1/2$.

(a) $1 + 3(1/2)$

(b) $13/2$

(c) $1 - \dfrac{1}{0 + 3(1)}$

(d) $1/3 + 1/2 - 1$.

(e) None of the above.

12. Differentiate $y = x^{x^3}$.

(a) $y' = (x^2 + 3x^2 \ln(x))\, x^{x^3}$

(b) $y' = x^3(x^{x^3 - 1})$

(c) $y' = 3x^2(x^{x^3})$

(d) $y' = x^2(3 + \ln(x))$

(e) None of the above.

13. Let $f(x) = x^4 + x^2 - 3$. Which of the following describes f in a small interval around $x = 1$?

(a) f is positive, concave up, and increasing.

(b) f is negative, concave up, and increasing.

(c) f is negative, concave down, and decreasing.

(d) f is stationary.

(e) None of the above.

14. Let $f(x) = x^5/20 + x^3/6 + 1$. What are the inflection points of f?

(a) $x = -1$

(b) $x = 0$ and $x = -1$

(c) $x = 0$

(d) The function has no inflection points.

(e) None of the above.

15. Let $f(x) = e^{x^2-x+1}$. What is the linearization of f through the point $(1, e)$?

(a) $L(x) = (2x-1)e^{x^2-x+1}(x-1) + e$

(b) $L(x) = e^{x-1} + e$.

(c) $L(x) = x - 1 + e$

(d) $L(x) = ex$

(e) None of the above

16. Suppose that the side lengths of an equilateral triangle are shrinking at a rate of 1 in/s, so that the figure is always an equilateral triangle. At the moment when the area of the triangle is $400\sqrt{3}$, at what rate is the area of the triangle changing? (You may use the fact that an equilateral triangle of side length s has area $\frac{\sqrt{3}s^2}{4}$.

(a) $-\frac{\sqrt{3}}{2}$ in$^2/s$.

(b) $-\frac{\sqrt{3}}{40}$ in$^2/s$.

(c) $-20\sqrt{3}$ in$^2/s$.

(d) $-4(400)^2\frac{\sqrt{3}}{3}$ in$^2/s$.

(e) None of the above.

17. Suppose that $2x\sin(y) = e^{xy}$. Find a formula for $\frac{dy}{dx}$ in terms of y and x.

(a) $\dfrac{dy}{dx} = \dfrac{e^x}{2\cos(y)}$

(b) $\dfrac{dy}{dx} = \dfrac{ye^{xy} + x\left(\frac{dy}{dx}\right)}{2\cos(y)}$

(c) $\dfrac{dy}{dx} = (xy)e^{xy} - 2\cos(y)$

(d) $\dfrac{dy}{dx} = \dfrac{ye^{xy} - 2\sin(y)}{-xe^{xy} + 2x\cos(y)}$

(e) None of the above

18. Compute $\int x^2 + 47x + e^x - \frac{1}{x}dx$.

(a) $x^3/3 + 47x^2/2 + e^x - \ln|x| + C$.

(b) $2x + 47 + e^x + 1/x^2 + C$

(c) $2x + 47 + e^x + 1/x^2$

(d) $x^3/3 + 47x^2/2 + e^x + \ln|x| + C$.

(e) None of the above

19. Which of the following is a formula for the Right Riemann sum of $f(x) = \sin(x)$ on the interval $[0, 1]$ with n subintervals of equal length?

(a) $\displaystyle\sum_{i=1}^{n} \frac{1}{n} \sin\left(\frac{i}{n}\right)$

(b) $\displaystyle\sum_{i=0}^{n-1} \frac{i+1}{n} \sin\left(\frac{i+1}{n}\right)$

(c) $\displaystyle\sum_{i=1}^{n-1} \frac{1}{n} \sin\left(\frac{i-1}{n}\right)$

(d) $\displaystyle\sum_{i=1}^{n} \frac{1}{n} \sin\left(\frac{1}{n}\right)$

(e) None of the above.

20. Compute $\frac{d}{dx} \arctan(e^x)$.

(a) $\sec^2(e^x)(e^x)$

(b) $\dfrac{e^x}{1 + e^{2x}}$

(c) $e^x \tan(e^x)$

(d) $\dfrac{\sec^2(e^x)}{e^x}$

(e) None of the above

21. Compute $\int -17x\cos(x^2)dx$.

(a) $(-17x^2/2)(\sin(x^2)) + C$

(b) $-17\cos(x^2) + (17)x\sin(x^2)(2x) + C$

(c) $(\frac{-17}{2})\sin(x^2) + C$.

(d) $\frac{-17\sin(x^2)}{x} + C$.

(e) None of the above.

22. Use the trapezoid rule with $n = 3$ trapezoids to approximate

$$\int_0^3 x^3 dx.$$

(a) $T_3 = (1/2)(0 + 1 + 8 + 27)$

(b) $T_3 = (1/2)(0 + 2 + 16 + 54)$

(c) $T_3 = (1/2)(0 + 2 + 16 + 27)$

(d) $T_3 = \frac{3^4}{4}$

(e) None of the above.

23. Write down the integral that you would need to compute in order to find the arc-length of the curve $y = e^x$ over the interval $[4, 7]$.

(a) $\int_4^7 e^x dx$

(b) $\int_4^7 1 + e^{2x} dx$

(c) $\int_4^7 \sqrt{1 + e^x} dx$

(d) $\int_4^7 \sqrt{1 + e^{2x}} dx$

(e) None of the above.

24. Find the area bounded between the curves $f(x) = 20x^3$ and $y = 20x$.

(a) -5

(b) 0

(c) 5

(d) 10

(e) None of the above.

25. A cannon on the ground fires a cannonball straight up into the air. If the acceleration due to gravity is $g = -9.8m/s^2$, and the cannonball takes 10 seconds to return to the ground, what is the initial velocity of the projectile. (Ignore air resistance).

(a) 9.8/2

(b) (9.8)5

(c) (9.8)10

(d) (9.8)50

(e) None of the above.

(for scratch work)