Final Examination

Math 3

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Name:

Instructor (circle):

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Instructions: You are not allowed to use calculators (or any kind of electronic equipment), books, or notes of any sort. It should go without saying, but we'll say it anyway, you are not to provide help to anyone nor receive help from anyone or any outside source. All of your answers must be marked on the Scantron form provided. Take a moment now to print your name and section clearly on your Scantron form and on this page of your exam booklet. You may write on the exam, but you will only receive credit for what you write on the Scantron form. At the end of the exam you must turn in both your Scantron form and your exam booklet. There are 30 multiple choice problems each worth 5 points. Check to see that you have 15 pages of questions plus the cover page.

1. For the function $f(x) = \dfrac{x^2}{x^2 - 1}$, its domain is

(a) $(-\infty, 1) \cup (1, \infty)$

(b) $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

(c) $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

(d) $(-\infty, -1) \cup (-1, 0) \cup (0, 1) \cup (1, \infty)$

(e) none of the above

2. For the same function $f(x)$ as in problem 1, its horizontal and vertical asymptotes are

(a) $y = 1, \ x = 1$

(b) $y = 1, \ x = 0, \ x = 1$

(c) $y = 1, \ x = -1, \ x = 1$

(d) $y = 0, \ x = -1, \ x = 0, \ x = 1$

(e) No single choice above gives the complete and correct list of horizontal and vertical asymptotes.

3. A function $f(x)$ has a vertical asymptote at $x = 0$, both from the left and the right. Which of the following lists of data about $f(x)$ cannot possibly be true?

(a) $f(x)$ is increasing on $(-\infty, 0)$ and on $(0, \infty)$,
   $f(x)$ is concave up on $(-\infty, 0)$ and on $(0, \infty)$.

(b) $f(x)$ is increasing on $(-\infty, 0)$ and on $(0, \infty)$,
   $f(x)$ is concave up on $(-\infty, 0)$ and concave down on $(0, \infty)$.

(c) $f(x)$ is increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$,
   $f(x)$ is concave up on $(-\infty, 0)$ and on $(0, \infty)$.

(d) $f(x)$ is neither increasing nor decreasing on $(-\infty, 0)$ and similarly on $(0, \infty)$,
   $f(x)$ is concave up on $(-\infty, 0)$ and concave down on $(0, \infty)$.

(e) Actually, each of the above is possible.

4. Find $\lim\limits_{h \to 0} \dfrac{\ln(2 + h) - \ln(2)}{h}$.

(a) It is $\ln(x)$.

(b) It is $1/x$.

(c) It is e^2.

(d) It is $1/2$.

(e) It doesn't exist.

5. For the function $f(x) = x^x$, its derivative is $f'(x) = x^x \ln(ex)$. The equation of the tangent line to the graph of $y = f(x)$ at $(1, 1)$ is

(a) $y = x$

(b) $y = ex - e + 1$

(c) $y = x^{x+1} \ln(ex)$

(d) Actually, there is no tangent line at this point.

(e) None of the above.

6. A baseball is popped straight up with an initial velocity of 40 meters per second. Assuming this occurred at sea level, that air resistance is negligible, and the acceleration due to gravity is 9.8 meters per second per second, the ball after 5 seconds is

(a) 200 meters high and still rising

(b) 49.5 meters high and falling

(c) 88 meters high and still rising

(d) 77.5 meters high and falling

(e) none of the above

7. Suppose $f(x)$ and $g(x)$ are functions with domain $(-\infty, \infty)$ and that $f(x)$ is even and $g(x)$ is odd. Which one of the following choices is not always true?

(a) $f(x)g(x)$ is odd

(b) $f(x) + g(x)^2$ is even

(c) $f(x)^2 + g(x)$ is odd

(d) $f(x) + 17$ is even

(e) $8g(x)$ is odd

8. Consider the functions $f(x) = x^2$, $g(x) = x^2 + x$, $h(x) = \arctan(x)$, all with domains $(-\infty, \infty)$. All of the following statements are true, except one of them. Pick out the false statement.

(a) $f(x)$ is not invertible

(b) $g(x)$ is not invertible

(c) $h(x)$ is invertible

(d) $f(x)$ restricted to $[-1, 0]$ is invertible

(e) $g(x)$ restricted to $[-1, 0]$ is invertible

9. Consider the function

$$f(x) = \begin{cases} \sin x, & \text{if } x \le \pi/4, \\ \cos x, & \text{if } x > \pi/4. \end{cases}$$

Which one of the following is true?

(a) $f(x)$ is continuous wherever it is defined.

(b) $f(x)$ is discontinuous at $x = \pi/4$ but has a removable discontinuity there.

(c) $f(x)$ is discontinuous at $x = \pi/4$ and the discontinuity there is not removable.

(d) $f(x)$ is differentiable (i.e., has a derivative) at $x = \pi/4$.

(e) none of the above

10. The absolute maximum of the function $f(x) = x^2 - x$ on the interval $[0, 1]$ occurs at

(a) $x = 1/2$

(b) $x = 1/\sqrt{2}$

(c) $x = 0$ and $x = 1$

(d) Actually, the function does not have an absolute maximum on $[0, 1]$.

(e) none of the above

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11. If one uses Simpson's method to approximate $\int_{-2}^{2} 2^{x^2} \, dx$ with $n = 4$, one gets

(a) 50/3

(b) 22

(c) 21

(d) 0

(e) none of the above

12. Compute $\lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \left(1 + \frac{i}{n}\right)^3$. It is

(a) 0.25

(b) 3.75

(c) 4

(d) 4/3

(e) none of the above

13. Which integral below has its value equal to the arclength from $x = 1$ to $x = 2$ along the curve $y = x^2 + x$?

(a) $\displaystyle\int_1^2 \sqrt{x^2 + x + 1}\,dx$

(b) $\displaystyle\int_1^2 \sqrt{x^4 + 2x^3 + x^2 + 1}\,dx$

(c) $\displaystyle\int_1^2 \sqrt{2x + 2}\,dx$

(d) $\displaystyle\int_1^2 \sqrt{4x^2 + 2}\,dx$

(e) $\displaystyle\int_1^2 \sqrt{4x^2 + 4x + 2}\,dx$

14. Compute the area of the figure bounded by the graphs of $y = x^3 - 2x$ and $y = x^2$. It is

(a) 0

(b) 27/12

(c) 37/12

(d) 3

(e) none of the above

15. The integral $\displaystyle\int (x^2 - 3)^2 \, dx$ is equal to

(a) $\dfrac{1}{3}(x^2 - 3)^3 + C$

(b) $\dfrac{1}{3}(x^2 - 3)^3(2x) + C$

(c) $\dfrac{1}{6}x^6 - \dfrac{3}{2}x^4 + 9x + C$

(d) $\dfrac{1}{5}x^5 - 2x^3 + 9x + C$

(e) none of the above

16. Which of the following sums is a lower Riemann sum (that is, a sum of areas of rectangles, each of which is an underestimate of an area) for the function $f(x) = x^3$ on the interval $[0, 2]$?

(a) $\dfrac{1}{n} \displaystyle\sum_{i=1}^{n} \left(\dfrac{2(i - 1)}{n} \right)^3$

(b) $\dfrac{1}{n} \displaystyle\sum_{i=1}^{n} \left(\dfrac{2i}{n} \right)^3$

(c) $\dfrac{2}{n} \displaystyle\sum_{i=1}^{n} \left(\dfrac{2(i - 1)}{n} \right)^3$

(d) $\dfrac{2}{n} \displaystyle\sum_{i=1}^{n} \left(\dfrac{2i}{n} \right)^3$

(e) none of the above

17. Evaluate $\dfrac{d}{dx} \displaystyle\int_x^5 \sin(t^3)\, dt$. It is

(a) $\sin(x^3)$

(b) $-\sin(x^3)$

(c) $\cos(x^3)$

(d) $3x^2 \cos(x^3)$

(e) $-3x^2 \cos(x^3)$

18. Recall that $\csc x = 1/\sin x$ and that $\cot x = \cos x/\sin x$. Evaluate $\displaystyle\int \dfrac{\cos x}{\sin^2 x}\, dx$. It is

(a) $\csc x + C$

(b) $\cot x + C$

(c) $-\csc x + C$

(d) $-\cot x + C$

(e) none of the above

19. Suppose that a population of mice grows exponentially. If 100 mice become 400 mice in one year, how many mice will there be in another 1.5 years?

(a) 800 mice

(b) 1600 mice

(c) 2400 mice

(d) 3200 mice

(e) 4800 mice

20. Let $y = x^{\ln x}$. Then $\dfrac{dy}{dx}$ is

(a) $(\ln x)x^{(\ln x)-1}$

(b) $(\ln x)x^{\ln x}$

(c) $x^{(\ln x)-1}$

(d) $2(\ln x)x^{(\ln x)-1}$

(e) none of the above

21. Evaluate $\dfrac{d}{dx} x^2 \arcsin x$. It is

(a) $x \arcsin x + \dfrac{x^2}{\sqrt{1-x^2}}$

(b) $x \arcsin x - \dfrac{x^2}{\sqrt{1-x^2}}$

(c) $2x \arcsin x + \dfrac{x^2}{\sqrt{1-x^2}}$

(d) $2x \arcsin x - \dfrac{x^2}{\sqrt{1-x^2}}$

(e) none of the above

22. Estimate $e^{0.1}$ using linearization. The estimate is

(a) 0.1

(b) 1.1

(c) $\dfrac{e}{10}$

(d) $1 + \dfrac{e}{10}$

(e) .105171

23. Evaluate $\lim\limits_{x\to\infty} \dfrac{x^2 + \cos x}{\sin x + 3x + 2x^2}$. It is

(a) 1/3

(b) 1/2

(c) diverges to ∞

(d) does not exist, but does not diverge to ∞

(e) none of the above

24. A spherical balloon is expanding at the rate of 10 cm³/s. How quickly is the radius increasing when the radius is equal to 10 cm? (Recall that a sphere of radius r has volume $\dfrac{4}{3}\pi r^3$).

(a) 4000π cm/s

(b) $\dfrac{1}{10\pi}$ cm/s

(c) $\dfrac{1}{40\pi}$ cm/s

(d) $\dfrac{1}{90\pi}$ cm/s

(e) none of the above

25. Which of the following statements accurately describes the function $f(x) = x^3 - 3x^2 - 24x + 16$?

   (a) $x = 4$ is a local maximum and $x = -2$ is also a local maximum

   (b) $x = 4$ is a local maximum and $x = -2$ is a local minimum

   (c) $x = 4$ is a local minimum and $x = -2$ is a local maximum

   (d) $x = 4$ is a local minimum and $x = -2$ is also a local minimum

   (e) none of the above

26. Which of the following accurately describes the quotient $\dfrac{f(a+h) - f(a)}{h}$?

   (a) The quotient equals the slope of the tangent line to $y = f(x)$ at $x = a$.

   (b) The quotient equals the slope of the tangent line to $y = f(x)$ at $x = a + h$.

   (c) The quotient equals the slope of the secant line passing through $(a, f(a))$ and $(a + h, f(a + h))$.

   (d) The quotient equals the slope of the secant line passing through $(a, f(a))$ and $(h, f(h))$.

   (e) none of the above

27. Suppose you have 12 hours to study for two exams, and you will spend x hours on exam A and y hours on exam B. You want to maximize xy^2. How should you allocate your study time?

(a) Study 0 hours for exam A, and 12 hours for exam B.

(b) Study 4 hours for exam A, and 8 hours for exam B.

(c) Study 6 hours for exam A, and 6 hours for exam B.

(d) Study 8 hours for exam A, and 4 hours for exam B.

(e) Don't study at all.

28. Which of the following is equal to $\int_{-1}^{1} \sin(x^2)\, dx$?

(a) $\int_{0}^{1} \sin(x^2)\, dx$

(b) 0

(c) $2 \int_{0}^{1} \sin(x^2)\, dx$

(d) $2\cos 1$

(e) none of the above

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29. Which of the following is a list of all inflection points of
$f(x) = \dfrac{x^5}{20} + \dfrac{x^3}{12} + 3x?$

(a) $x = -1$

(b) $x = -1, 0$

(c) $x = 0$

(d) $x = -1, 1$

(e) none of the above

30. Solve the initial value problem $\dfrac{dy}{dx} = 2x \sec y$, $y(0) = 0$.

(a) $y = \sin x$

(b) $y = \arcsin x$

(c) $y = -\arcsin(x^2)$

(d) $y = -\arcsin x$

(e) $y = \arcsin(x^2)$