Extra practice: Related rates

1. Relate the change of the volume of a sphere of radius r versus time t to the change in the radius with respect to time.

2. Find the rate of change of the volume of a cylinder of radius r and height h with respect to a change in the radius, assuming the height is also a function of r.

3. Find the rate of change of the curved surface of a cone of radius r and height h with respect to a change in the radius, assuming the height is also a function of r.

4. The side of a square is increasing at the rate of 0.2 cm/s. Find the rate of increase of the perimeter of the square.

5. A balloon which always remains spherical is being inflated by pumping in 900 cubic centimeters of gas per second. Find the rate at which the radius of the balloon is increasing when the radius is 15cm.

6. The surface area of a spherical bubble is increasing at $2 \text{ cm}^2/\text{s}$. When the radius of the bubble is 6 cm at what rate is the volume of the bubble increasing?

7. The bottom of a rectangular swimming pool is 25×40 meters. Water is pumped into the tank at the rate of 500 cubic meters per minute. Find the rate at which the level of the water in the tank is rising.

8. A streetlight is at the top of a 15 foot tall pole. A man 6 ft tall walks away from the pole with a speed of 5 ft/s along a straight path.
   (a) How fast is the tip of his shadow moving when he is 40 feet from the pole?
   (b) How fast is his shadow lengthening at that point?

9. A lighthouse is on a small island 3 km away from the nearest point P on a straight shoreline and its light turns four revolutions per minute. How fast is the beam of light moving along the shoreline when it is 1 km from P?

10. A boat is pulled into a dock by a rope attached to the bow of the boat a passing through a pulley on the dock that is 1 m higher than the bow of the boat. If the rope is pulled in at a rate of 1 m/s how fast is the boat approaching the dock when it is 8 m from the dock?

11. Gravel is being dumped from a conveyor belt at a rate of $30 \text{ ft}^3/\text{min}$ and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 10 ft high?

12. Water is dripping from a tiny hole in the vertex in the bottom of a conical funnel at a uniform rate of 4 cm^3/s. When the slant height of the water is 3 cm, find the rate of decrease of the slant height of the water, given that the vertical angle of the funnel is 120°.

13. Water is leaking out of an inverted conical tank at a rate of 10,000 cm^3/min at the same time that water is being pumped into the tank at a constant rate. The tank has height 6 m and the diameter at the top is 4 m. If the water height is rising at a rate of 20 cm/min when the hieght of the water is 2 m, find the rate at which water is being pumped into the tank.

14. Oil is leaking from a cylindrical drum at a rate of 16 milliliters per second. If the radius of the drum is 7 cm and its height is 60 cm find the rate at which the level of oil is changing when the oil level is 18 cm.

15. A ladder 10 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a speed of 2 ft/s, how fast is the angle between the top of the ladder and the wall changing when the angle is $\pi/4$ radians?

16. A ladder 13 meters long is leaning against a wall. The bottom of the ladder is pulled along the ground away from from the wall at the rate of 2 m/s. How fast is its height on the wall decreasing when the foot of the ladder is 5 m away from the wall?

17. A man is moving away from a 40 meter tower at a speed of 2 m/s. Find the rate at which the angle of elevation of the top of the tower is changing when he is at a distance of 30 meters from the foot of the tower. Assume that the eye level of the man is 1.6 meters from the ground.

18. Find the angle which increases twice as fast as its sine.

19. a rocket launching pad. A rocket rises vertically and its speed is 600 ft/s when it has risen 3000 feet.

    (a) How fast is the distance from the television camera to the rocket changing at that moment?
    (b) How fast is the camera's angle of elevation changing at that same moment?

1. According to the US Census, the world population P, in billions, was approximately

$$P = 6.7e^{0.011t},$$

where t is in years since January 1, 2007. At what rate was the world's population increasing on that date? Give your answer in millions of people per year. [The world population growth rate has actually decreased since 2007.]

2. Atmospheric pressure decays exponentially as altitude increases. With pressure, P, in inches of mercury and altitude, h, in feet above sea level, we have

$$P = 30e^{-3.23 \times 10^{-5}h}.$$

(a) At what altitude is the atmospheric pressure 25 inches of mercury?

(b) A glider measures the pressure to be 25 inches of mercury and experiences a pressure increase of 0.1 inches of mercury per minute. At what rate is it changing altitude?

3. The Dubois formula relates a person's surface area, s, in meters^2, to weight, w, in kg, and height,t h, in cm, by

$$s = 0.01w^{0.25}h^{0.75}.$$

(a) What is the surface area of a person who weighs 60 kg and is 150 cm tall?

(b) The person in part (a) stays constant height but increases in weight by 0.5 kg/year. At what rate is his surface area increasing when his weight is 62 kg?

4. With time, t, in minutes, the temperature, H, in degrees Celsius, of a bottle of water put in the refrigerator at $t = 0$ is given by

$$H = 4 + 16e^{-0.02t}.$$

How fast is the water cooling initially? After 10 minutes? Give units.

5. The power, P, dissipated when a 9-volt battery is put across a resistance of R ohms is given by

$$P = \frac{81}{R}.$$

What is the rate of change of power with respect to resistance?

6. With length, l, in meters, the period T, in seconds, of a pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{9.8}}.$$

(a) How fast does the period increase as l increases?

(b) Does this rate of change increase or decrease as l increases?

7. At time t, in hours, a lake is covered with ice of thickness y cm, where $y = 0.2t^{1.5}$.

(a) How fast is the ice forming when $t = 1$? When $t = 2$? Give units.

(b) If ice forms for $0 \le t \le 3$, at what time in this interval is the ice thickest? At what time is the ice forming fastest?

8. A dose, D, of a drug causes a temperature change, T, in a patient. For C a positive constant, T is given by

$$T = \left(\frac{C}{2} - \frac{D}{3}\right)D^3.$$

(a) What is the rate of change of temperature change with respect to dose?

(b) For what doses does the temperature change increase as the dose increases?

9. For positive constants k and g, the velocity, v, of a particle of mass m at time t is given by

$$v = \frac{mg}{k}\left(1 - e^{-kt/m}\right).$$

At what rate is the velocity changing at time 0? At $t = 1$? What do your answers tell you about the motion?

10. The average cost per item, C, in dollars, of manufacturing a quantity q of cell phones is given by

$$C = \frac{a}{q} + b \qquad \text{where } a, b \text{ are positive constants.}$$

(a) Find the rate of change of C as q increases. What are its units?

(b) If production increases at a rate of 100 cell phones per week, how fast is the average cost changing? Is the average cost increasing or decreasing?

11. For positive constants A and B, the force, F, between two atoms in a molecule at a distance r apart is given by

$$F = -\frac{A}{r^2} + \frac{B}{r^3}.$$

(a) How fast does force change as r increases? What type of units does it have?

(b) If at some time t the distance is changing at a rate k, at what rate is the force changing with time? What type of units does this rate of change have?

12. An item costs $500 at time $t = 0$ and costs P in year t. When inflation is $r\%$ per year, the price is given by

$$P = 500e^{rt/100}.$$

(a) If r is a constant, at what rate is the price rising (in dollars per year)

(i) Initially?   (ii) After 2 years?

(b) Now suppose that r is increasing by 0.3 per year when $r = 4$ and $t = 2$. At what rate (dollars per year) is the price increasing at that time?

13. A voltage V across a resistance R generates a current

$$I = \frac{V}{R}.$$

If a constant voltage of 9 volts is put across a resistance that is increasing at a rate of 0.2 ohms per second when the resistance is 5 ohms, at what rate is the current changing?

14. The gravitational force, F, on a rocket at a distance, r, from the center of the earth is given by

$$F = \frac{k}{r^2},$$

where $k = 10^{13}$ newton $\cdot$ km^2. When the rocket is 10^4 km from the center of the earth, it is moving away at 0.2 km/sec. How fast is the gravitational force changing at that moment? Give units. (A newton is a unit of force.)

15. The potential, ϕ, of a charge distribution at a point on the positive x-axis is given by

$$\phi = 2\pi\left(\sqrt{x^2 + 4} - x\right)$$

where x is in centimeters. A particle at $x = 3$ is moving to the left at a rate of 0.2 cm/sec. At what rate is its potential changing?

16. A pyramid has height h and a square base with side x. The volume of a pyramid is $V = \frac{1}{3}x^2 h$. If the height remains fixed and the side of the base is decreasing by 0.002 meter/yr, what rate is the volume decreasing when the height is 120 meters and the width is 150 meters?

17. If θ is the angle between a line through the origin and the positive x-axis, the area, in cm^2, of part of a rose petal is

$$A = \frac{9}{16}(4\theta - \sin(4\theta)).$$

If the angle θ is increasing at a rate of 0.2 radians per minute, at what rate is the area changing when $\theta = \pi/4$?

18. A thin uniform rod of length l cm and a small particle lie on a line separated by a distance of a cm. If K is a positive constant and F is measured in newtons, the gravitational force between them is

$$F = \frac{K}{a(a + l)}.$$

(a) If a is increasing at the rate 2 cm/min when $a = 15$ and $l = 5$, how fast is F decreasing?

(b) If l is decreasing at the rate 2 cm/min when $a = 15$ and $l = 5$, how fast is F increasing?

19. The depth of soot deposited from a smokestack is given by $D = K(r+1)e^{-r}$, where r is the distance from the smokestack. What is the relationship between the rate of change of r with respect to time and the rate of change of D with respect to time?

20. The mass of a circular oil slick of radius r is $M = K(r - \ln(1+r))$, where K is a positive constant. What is the relationship between the rate of change of the radius with respect to t and the rate of change of the mass with respect to time?

21. Gasoline is pouring into a cylindrical tank of radius 3 feet. When the depth of the gasoline is 4 feet, the depth is increasing at 0.2 ft/sec. How fast is the volume of gasoline changing at that instant?

22. The metal frame of a rectangular box has a square base. The horizontal rods in the base are made out of one metal and the vertical rods out of a different metal. If the horizontal rods expand at a rate of 0.001 cm/hr and the vertical rods expand at a rate of 0.002 cm/hr, at what rate is the volume of the box expanding when the base has an area of $9\,\mathrm{cm}^2$ and the volume is $180\,\mathrm{cm}^3$?

23. Point P moves around the unit circle.[8] (See Figure 4.82.) The angle θ, in radians, changes with time as shown in Figure 4.83.

(a) Estimate the coordinates of P when $t = 2$.
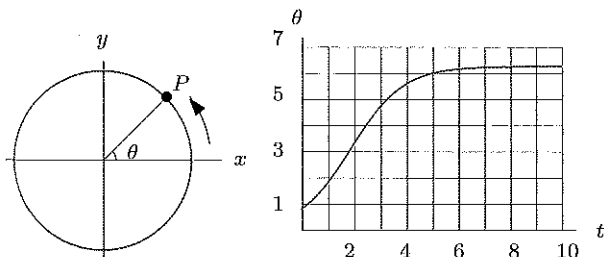(b) When $t = 2$, approximately how fast is the point P moving in the x-direction? In the y-direction?

Figure 4.82   Figure 4.83

24. Figure 4.84 shows the number of gallons, G, of gasoline used on a trip of M miles.

(a) The function f is linear on each of the intervals $0 < M < 70$ and $70 < M < 100$. What is the slope of these lines? What are the units of these slopes?
(b) What is gas consumption (in miles per gallon) during the first 70 miles of this trip? During the next 30 miles?
(c) Figure 4.85 shows distance traveled, M (in miles), as a function of time t, in hours since the start of

the trip. Describe this trip in words. Give a possible explanation for what happens one hour into the trip. What do your answers to part (b) tell you about the trip?

(d) If we let $G = k(t) = f(h(t))$, estimate $k(0.5)$ and interpret your answer in terms of the trip.
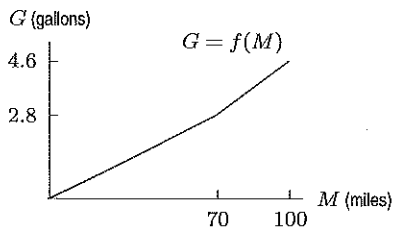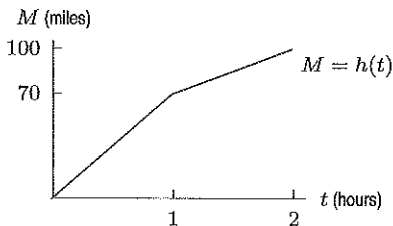(e) Find $k'(0.5)$ and $k'(1.5)$. Give units and interpret your answers.

Figure 4.84

Figure 4.85:

25. On February 16, 2007, paraglider Eva Wisnierska[9] was caught in a freak thunderstorm over Australia and carried upward at a speed of about 3000 ft/min. Table 4.4 gives the temperature at various heights. Approximately how fast (in °F/ per minute) was her temperature decreasing when she was at 4000 feet?

Table 4.4

y (thousand ft)	2	4	6	8	10	12	14	16
H (°F)	60	52	38	31	23	16	9	2

26. The circulation time of a mammal (that is, the average time it takes for all the blood in the body to circulate once and return to the heart) is proportional to the fourth root of the body mass of the mammal. The constant of proportionality is 17.40 if circulation time is in seconds and body mass is in kilograms. The body mass of a growing child is 45 kg and is increasing at a rate of 0.1 kg/month. What is the rate of change of the circulation time of the child?

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[8]Based on an idea from Caspar Curjel.
[9]www.sciencedaily.com, accessed May 15, 2007.

27. Coroners estimate time of death using the rule of thumb that a body cools about 2°F during the first hour after death and about 1°F for each additional hour. Assuming an air temperature of 68°F and a living body temperature of 98.6°F, the temperature $T(t)$ in °F of a body at a time t hours since death is given by

$$T(t) = 68 + 30.6e^{-kt}.$$

(a) For what value of k will the body cool by 2°F in the first hour?

(b) Using the value of k found in part (a), after how many hours will the temperature of the body be decreasing at a rate of 1°F per hour?

(c) Using the value of k found in part (a), show that, 24 hours after death, the coroner's rule of thumb gives approximately the same temperature as the formula.

28. A certain quantity of gas occupies a volume of $20\ \text{cm}^3$ at a pressure of 1 atmosphere. The gas expands without the addition of heat, so, for some constant k, its pressure, P, and volume, V, satisfy the relation

$$PV^{1.4} = k.$$

(a) Find the rate of change of pressure with volume. Give units.

(b) The volume is increasing at $2\ \text{cm}^3/\text{min}$ when the volume is $30\ \text{cm}^3$. At that moment, is the pressure increasing or decreasing? How fast? Give units.

29. (a) A hemispherical bowl of radius 10 cm contains water to a depth of h cm. Find the radius of the surface of the water as a function of h.

(b) The water level drops at a rate of 0.1 cm per hour. At what rate is the radius of the water decreasing when the depth is 5 cm?

30. A cone-shaped coffee filter of radius 6 cm and depth 10 cm contains water, which drips out through a hole at the bottom at a constant rate of $1.5\ \text{cm}^3$ per second.

(a) If the filter starts out full, how long does it take to empty?

(b) Find the volume of water in the filter when the depth of the water is h cm.

(c) How fast is the water level falling when the depth is 8 cm?

31. A spherical snowball is melting. Its radius is decreasing at 0.2 cm per hour when the radius is 15 cm. How fast is its volume decreasing at that time?

32. A ruptured oil tanker causes a circular oil slick on the surface of the ocean. When its radius is 150 meters, the radius of the slick is expanding by 0.1 meter/minute and its thickness is 0.02 meter. At that moment:

(a) How fast is the area of the slick expanding?

(b) The circular slick has the same thickness everywhere, and the volume of oil spilled remains fixed. How fast is the thickness of the slick decreasing?

33. A potter forms a piece of clay into a cylinder. As he rolls it, the length, L, of the cylinder increases and the radius, r, decreases. If the length of the cylinder is increasing at 0.1 cm per second, find the rate at which the radius is changing when the radius is 1 cm and the length is 5 cm.

34. A gas station stands at the intersection of a north-south road and an east-west road. A police car is traveling toward the gas station from the east, chasing a stolen truck which is traveling north away from the gas station. The speed of the police car is 100 mph at the moment it is 3 miles from the gas station. At the same time, the truck is 4 miles from the gas station going 80 mph. At this moment:

(a) Is the distance between the car and truck increasing or decreasing? How fast? (Distance is measured along a straight line joining the car and the truck.)

(b) How does your answer change if the truck is going 70 mph instead of 80 mph?

35. A train is traveling at 0.8 km/min along a long straight track, moving in the direction shown in Figure 4.86. A movie camera, 0.5 km away from the track, is focused on the train.
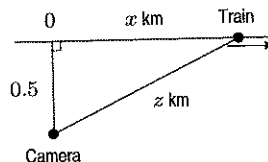
(a) Express z, the distance between the camera and the train, as a function of x.

(b) How fast is the distance from the camera to the train changing when the train is 1 km from the camera? Give units.

(c) How fast is the camera rotating (in radians/min) at the moment when the train is 1 km from the camera?

Figure 4.86

36. A lighthouse is 2 km from the long, straight coastline shown in Figure 4.87. Find the rate of change of the distance of the spot of light from the point O with respect to the angle θ.
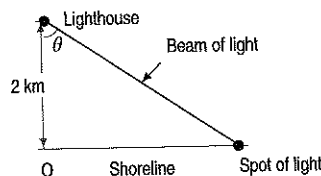

Figure 4.87

37. A train is heading due west from St. Louis. At noon, a plane flying horizontally due north at a fixed altitude of 4 miles passes directly over the train. When the train has traveled another mile, it is going 80 mph, and the plane has traveled another 5 miles and is going 500 mph. At that moment, how fast is the distance between the train and the plane increasing?

38. The radius of a spherical balloon is increasing by 2 cm/sec. At what rate is air being blown into the balloon at the moment when the radius is 10 cm? Give units in your answer.

39. A spherical cell is growing at a constant rate of $400 \ \mu m^3$/day ($1 \ \mu m = 10^{-6}$ m). At what rate is its radius increasing when the radius is $10 \ \mu m$?

40. A raindrop is a perfect sphere with radius r cm and surface area S cm^2. Condensation accumulates on the raindrop at a rate equal to kS, where $k = 2$ cm/sec. Show that the radius of the raindrop increases at a constant rate and find that rate.

41. The length of each side of a cube is increased at a constant rate. Which is greater, the relative rate of change of the volume of the cube, $\dfrac{1}{V}\dfrac{dV}{dt}$, or the relative change of the surface area of the cube, $\dfrac{1}{A}\dfrac{dA}{dt}$?

42. Sand falls from a hopper at a rate of 0.1 cubic meters per hour and forms a conical pile beneath. If the side of the cone makes an angle of $\pi/6$ radians with the vertical, find the rate at which the height of the cone increases. At what rate does the radius of the base increase? Give both answers in terms of h, the height of the pile in meters.

43. A circular region is irrigated by a 20 meter long pipe, fixed at one end and rotating horizontally, spraying water. One rotation takes 5 minutes. A road passes 30 meters from the edge of the circular area. See Figure 4.88.

(a) How fast is the end of the pipe, P, moving?
(b) How fast is the distance PQ changing when θ is $\pi/2$? When θ is 0?
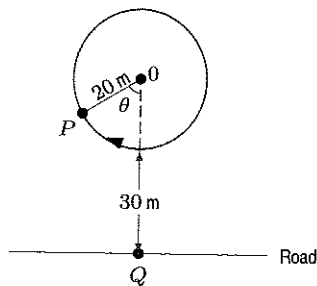

Figure 4.88

44. A water tank is in the shape of an inverted cone with depth 10 meters and top radius 8 meters. Water is flowing into the tank at 0.1 cubic meters/min but leaking out at a rate of $0.004h^2$ cubic meters/min, where h is the depth of the water in meters. Can the tank ever overflow?

45. For the amusement of the guests, some hotels have elevators on the outside of the building. One such hotel is 300 feet high. You are standing by a window 100 feet above the ground and 150 feet away from the hotel, and the elevator descends at a constant speed of 30 ft/sec, starting at time $t = 0$, where t is time in seconds. Let θ be the angle between the line of your horizon and your line of sight to the elevator. (See Figure 4.89.)

(a) Find a formula for $h(t)$, the elevator's height above the ground as it descends from the top of the hotel.
(b) Using your answer to part (a), express θ as a function of time t and find the rate of change of θ with respect to t.
(c) The rate of change of θ is a measure of how fast the elevator appears to you to be moving. At what height is the elevator when it appears to be moving fastest?
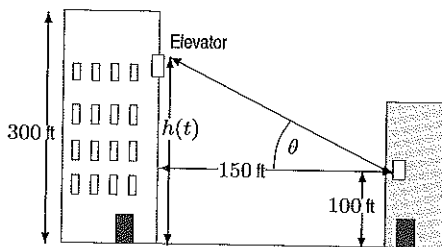

Figure 4.89

46. In a romantic relationship between Angela and Brian, who are unsuited for each other, $a(t)$ represents the affection Angela has for Brian at time t days after they meet, while $b(t)$ represents the affection Brian has for Angela at time t. If $a(t) > 0$ then Angela likes Brian; if $a(t) < 0$ then Angela dislikes Brian; if $a(t) = 0$ then Angela neither likes nor dislikes Brian. Their affection for each other is given by the relation $a^2(t) + b^2(t) = c$, where c is a constant.

(a) Show that $a(t) \cdot a'(t) = -b(t) \cdot b'(t)$.
(b) At any time during their relationship, the rate per day at which Brian's affection for Angela changes is $b'(t) = -a(t)$. Explain what this means if Angela

    (i) Likes Brian,   (ii) Dislikes Brian.

(c) Use parts (a) and (b) to show that $a'(t) = b(t)$. Explain what this means if Brian

    (i) Likes Angela,   (ii) Dislikes Angela.

(d) If $a(0) = 1$ and $b(0) = 1$ who first dislikes the other?

Extra practice: Related rates

1. Relate the change of the volume of a sphere of radius r versus time t to the change in the radius with respect to time.

Since $V = \frac{4}{3}\pi r^3$,

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt}$$
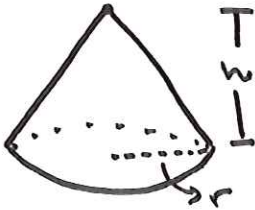
$$= 4\pi r^2 \frac{dr}{dt}$$

2. Find the rate of change of the volume of a cylinder of radius r and height h with respect to a change in the radius, assuming the height is also a function of r.
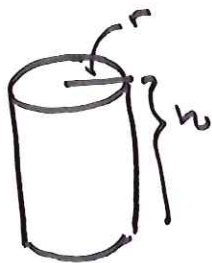
$$V = \pi r^2 h$$

So

$$\frac{dV}{dr} = \pi \left(2rh + r^2 \frac{dh}{dt} \right)$$

3. Find the rate of change of the curved surface of a cone of radius r and height h with respect to a change in the radius, assuming the height is also a function of r.

$$S = \pi r \sqrt{r^2 + h^2}$$

so $\quad \boxed{\dfrac{dS}{dr} = \pi \sqrt{r^2+h^2} + \pi r \cdot \frac{1}{2}(r^2+h^2)^{-1/2}\left(2r + 2h\frac{dh}{dr}\right)}$

4. The side of a square is increasing at the rate of 0.2 cm/s. Find the rate of increase of the perimeter of the square.

$\}\, s$
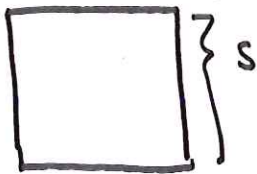
$\dfrac{ds}{dt} = \dfrac{1}{5}$, what is $\dfrac{dP}{dt}$?

$P = 4s$. So $\dfrac{dP}{dt} = 4\dfrac{ds}{dt}$

$= 4 \cdot \dfrac{1}{5} = \boxed{.8}$ cm/s

5. A balloon which always remains spherical is being inflated by pumping in 900 cubic centimeters of gas per second. Find the rate at which the radius of the balloon is increasing when the radius is 15cm.
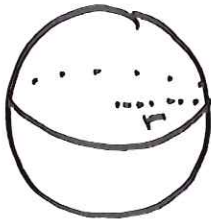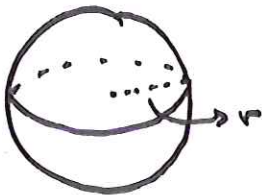
$$V = \frac{4}{3}\pi r^3. \qquad \text{what is} \quad \frac{dr}{dt}\Big|_{r=15} ?$$

$$\frac{dv}{dt} = 900$$

$$\frac{dv}{dt} = \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\text{So} \quad \frac{dr}{dt} = \frac{dv}{dt} \cdot \frac{1}{4\pi r^2} = \frac{900}{4\pi r^2} \Rightarrow \frac{dr}{dt}\Big|_{r=15} = \boxed{\frac{900}{4\pi(15)^2}}$$

$$cm/s$$

6. The surface area of a spherical bubble is increasing at 2 cm²/s. When the radius of the bubble is 6 cm at what rate is the volume of the bubble increasing?

$$S = 4\pi r^2 \qquad V = \frac{4}{3}\pi r^3$$

$$\text{if} \quad \frac{dS}{dt} = 2, \quad \text{what is} \quad \frac{dV}{dt}\Big|_{r=6} ?$$

$$\frac{dS}{dt} = 4\pi \cdot 2r \cdot \frac{dr}{dt} \qquad\qquad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\text{So} \quad \frac{dr}{dt} = \frac{1}{8\pi r}\frac{dS}{dt} = \frac{1}{8\pi r} \cdot 2 \qquad \text{So} \quad \frac{dV}{dt}\Big|_{r=6} = 4\pi 6^2 \cdot \frac{1}{24\pi}$$
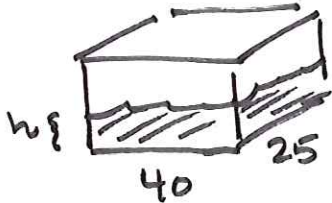
$$\text{So} \quad \frac{dr}{dt}\Big|_{r=6} = \frac{2}{8\pi \cdot 6} = \frac{1}{24\pi} \qquad\qquad = \boxed{6}\ cm^3/s$$

7. The bottom of a rectangular swimming pool is 25×40 meters. Water is pumped into the tank at the rate of 500 cubic meters per minute. Find the rate at which the level of the water in the tank is rising.
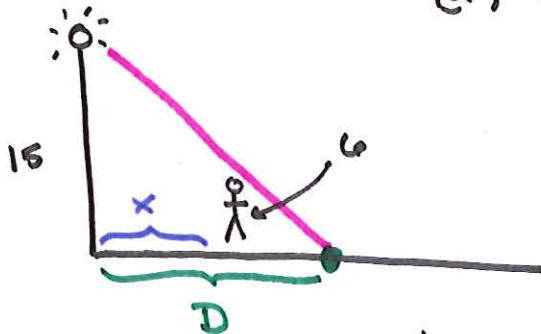
$$V = h * 40 * 25$$
$$= 1000 \, h$$

So $\dfrac{dV}{dt} = 1000 \dfrac{dh}{dt}$
$\hookrightarrow 500$

So $\dfrac{dh}{dt} = 500 * \dfrac{1}{1000} = \boxed{\dfrac{1}{2}} \dfrac{m}{min}$

8. A streetlight is at the top of a 15 foot tall pole. A man 6 ft tall walks away from the pole with a speed of 5 ft/s along a straight path.

(a) How fast is the tip of his shadow moving when he is 40 feet from the pole?

(b) How fast is his shadow lengthening at that point?

(a) call: D = distance btwn tip of shadow & lamp.

X = distance btwn guy & lamp.

Similar triangles says:

$$\dfrac{D}{15} = \dfrac{D-x}{6}$$

So $15x = (15-6)D = 9D$

So $15\dfrac{dx}{dt} = 9\dfrac{dD}{dt}$

$15 \cdot 5 =$

So $\dfrac{dD}{dt} = \boxed{\dfrac{75}{9}}$ f/s

(b) $\ell = D - x$

So $\dfrac{d\ell}{dt} = \dfrac{dD}{dt} - \dfrac{dx}{dt}$

$= \boxed{-5 + \dfrac{75}{9}}$

$= \boxed{30/9}$ f/s

9. A lighthouse is on a small island 3 km away from the nearest point P on a straight shoreline and its light turns four revolutions per minute. How fast is the beam of light moving along the shoreline when it is 1 km from P?
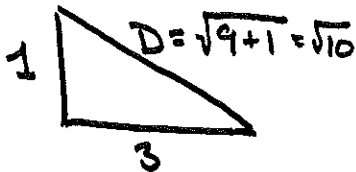

$$\frac{d\theta}{dt} = 4 * 2\pi \qquad \text{what is } \left.\frac{d\ell}{dt}\right|_{\ell=1} ?$$

relate θ & ℓ:

$$\tan\theta = \ell/3 \qquad \text{so} \qquad \sec^2\theta \frac{d\theta}{dt} = \frac{1}{3}\frac{d\ell}{dt}$$

when $\ell=1$:

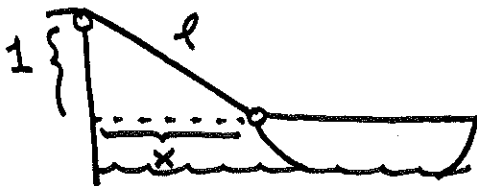$D = \sqrt{9+1} = \sqrt{10}$

$$\longrightarrow \text{so} \quad \sec\theta|_{\ell=1} = \frac{\sqrt{10}}{3}$$
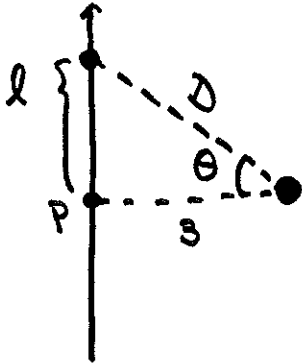
$$\text{so} \quad \left.\frac{d\ell}{dt}\right|_{\ell=1} = \left(\frac{\sqrt{10}}{3}\right)^2 \cdot 4 \cdot 2\pi \cdot 3 = \boxed{\frac{10}{3} \cdot 8\pi}$$

10. A boat is pulled into a dock by a rope attached to the bow of the boat a passing through a pulley on the dock that is 1 m higher than the bow of the boat. If the rope is pulled in at a rate of 1 m/s how fast is the boat approaching the dock when it is 8 m from the dock?

$$\text{what is } \left.\frac{dx}{dt}\right|_{x=8} ?$$

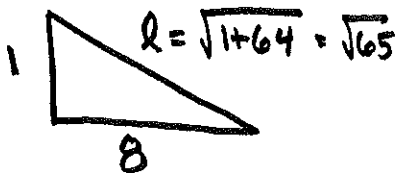$$\frac{d\ell}{dt} = -1 .$$

relate x & ℓ:

$$1^2 + x^2 = \ell^2, \qquad \text{so} \quad 2x\frac{dx}{dt} = 2\ell\frac{d\ell}{dt} = -2\ell$$

$$\text{so} \quad \frac{dx}{dt} = -\frac{\ell}{x}$$

when $x=8$:

$\ell = \sqrt{1+64} = \sqrt{65}$

$$\text{So} \quad \frac{dx}{dt} = -\frac{\sqrt{65}}{8} \quad \Rightarrow \quad \text{boat is } \text{mov}$$

$$\sqrt{65}/8 \text{ mps.}$$

11. Gravel is being dumped from a conveyor belt at a rate of 30 ft³/min and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 10 ft high?

$h = 2r$

$V = \frac{\pi}{3} r^2 h = \frac{\pi}{3} \left(\frac{1}{2}h\right)^2 \cdot h = \frac{\pi}{12} h^3$

$\frac{dV}{dt} = 30$. what is $\frac{dh}{dt}\Big|_{h=10}$?

$30 = \frac{dV}{dt} = \frac{\pi}{12} \cdot 3h^2 \frac{dh}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$. So $\frac{dh}{dt} = \frac{30 \cdot 4}{\pi \cdot h^2}$

So $\frac{dh}{dt}\Big|_{h=10} = \frac{30 \cdot 4}{\pi \cdot 100} = \frac{12}{\pi \cdot 10} = \boxed{\frac{6}{\pi \cdot 5}}$

12. Water is dripping from a tiny hole in the vertex in the bottom of a conical funnel at a uniform rate of 4 cm³/s. When the slant height of the water is 3 cm, find the rate of decrease of the slant height of the water, given that the vertical angle of the funnel is 120°.
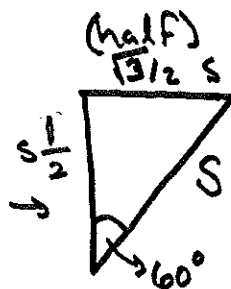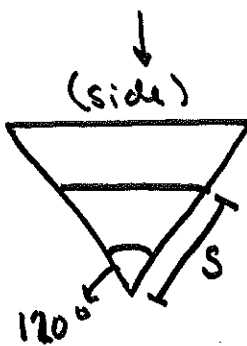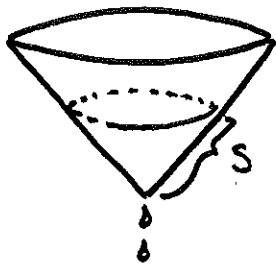
$\frac{dV}{dt} = -4$. what is $\frac{ds}{dt}\Big|_{s=3}$

$V = \frac{\pi}{3} r^2 h = \frac{\pi}{3}\left(\frac{\sqrt{3}}{2}s\right)^2 \cdot \frac{s}{2} = \frac{\pi}{8} s^3$

(side)

(half)
$\frac{\sqrt{3}}{2} s$

$s\frac{1}{2}$

s S

120°   →60°

so
$r = \frac{\sqrt{3}}{2} s$

$h = \frac{1}{2}s$

So $\frac{dV}{dt} = \frac{\pi}{8} \cdot 3 s^2 \frac{ds}{dt}$

so $\frac{ds}{dt} = \frac{8}{3\pi s^2} \frac{dV}{dt}$

So $\frac{ds}{dt}\Big|_{s=3} = \boxed{\frac{8}{\pi \cdot 27} \cdot (-4)}$

(answers to scanned exercises)

1 73.7 m/yr

3 (a) 1.19 meter^2
   (b) 0.0024 meter^2/year

5 $-81/R^2$

7 (a) 0.3 cm/hr; 0.424 cm/hr
   (b) $t = 3$; $t = 3$

9 g; $ge^{-k/m}$; acceleration decreases from g at $t = 0$

11 (a) $2A/r^3 - 3B/r^4$; units of force/units of distance
   (b) $k(2A/r^3 - 3B/r^4)$; units of force/units of time

13 -0.072 amps/sec

15 0.211 units/sec

17 0.9 cm^2/min

19 $dD/dt = -Kre^{-r}dr/dt$

21 5.655 ft^3/sec

23 (a) $(-0.99, -0.16)$
   (b) $v_x = 0.32$
         $v_y = -1.98$

25 Between $12°$F/min and $20°$F/min

27 (a) $k \approx 0.067$
   (b) $t \approx 10.3$ hours
   (c) Formula:
         $T(24) \approx 74.1°$F,
         rule of thumb: $73.6°$F

29 (a) $r = \sqrt{20h - h^2}$ cm
   (b) $-1/(2\sqrt{75}) = 0.0577$ cm/hr

31 $180\pi = 565.487$ cm^3/hr

33 -0.01 cm/sec

35 (a) $z = \sqrt{0.25 + x^2}$
   (b) 0.693 km/min
   (c) 0.4 radians/min

37 398.103 mph

39 $1/\pi \approx 0.32 \mu$m/day

41 $(1/V)dV/dt$

43 (a) 25.133 meters/min
   (b) 23.335 meters/min; 0 meters/min

45 (a) $h(t) = 300 - 30t$
         $0 \le t \le 10$
   (b) $\theta = \arctan((200 - 30t)/150)$
         $d\theta/dt =$
         $-\left(\frac{1}{5}\right)\left(\frac{150^2}{150^2 + (200-30t)^2}\right)$
   (c) When elevator is at level of observer