Related rates

Example

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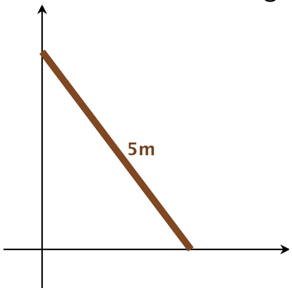

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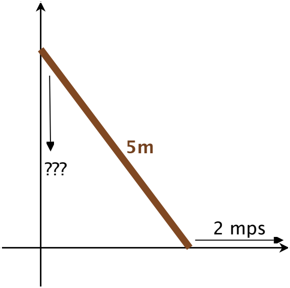

Move the base out at 2 m/s

How fast does the top move down the wall?

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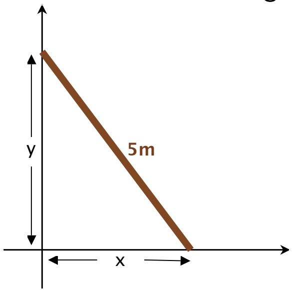

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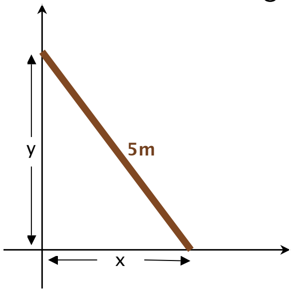

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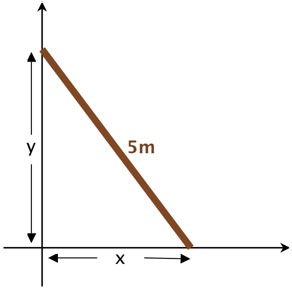

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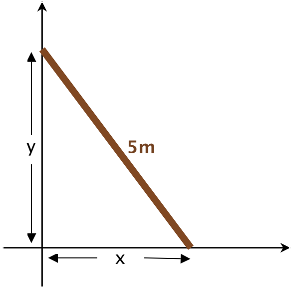

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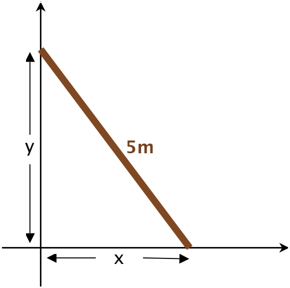

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Notice:
(1) $\frac{dy}{dt} < 0$ (y is decreasing) and (2) $\lim_{x \to 5^-} \frac{dy}{dt} \to -\infty$

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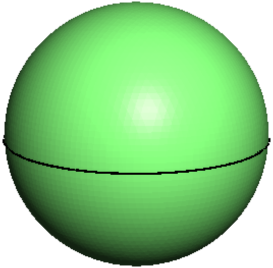
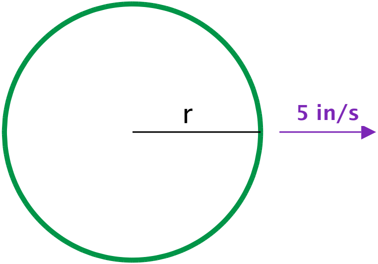
Suppose you have a sphere whose radius is growing at a rate of 5in/s. How fast is the volume growing when the radius is 3in?





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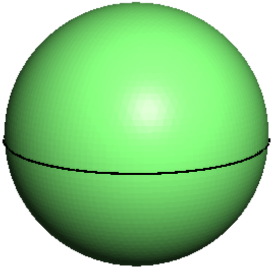

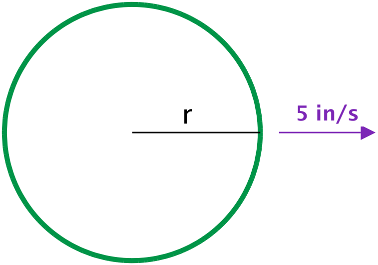

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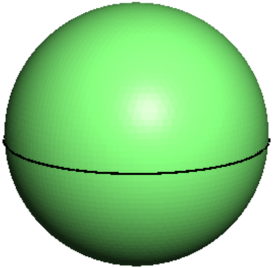

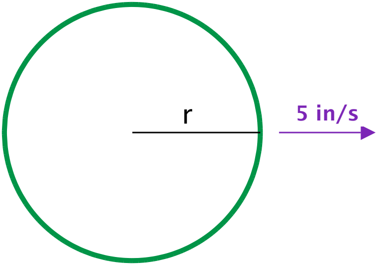

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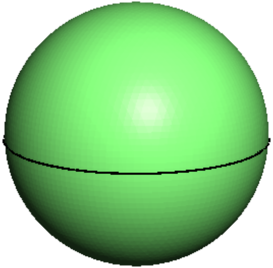

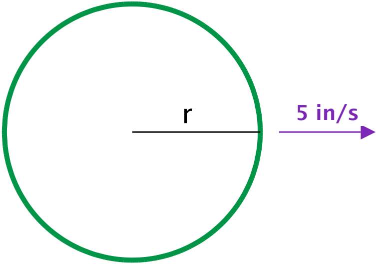

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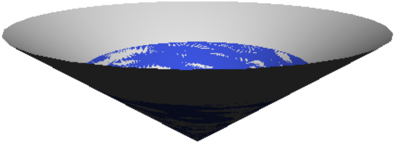
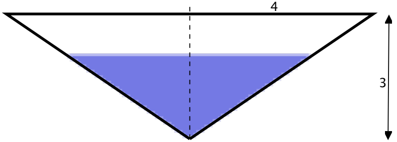
Substitute in the known values:
$\frac{dV}{dt} = 4\pi * 3^2 * 5 = \boxed{4 * 9 * 5\pi \text{ in}^3/\text{s}}$

Take an upside-down cone-shaped bowl, with a radius of 4in at the top and a total height of 3in fill it with water at a rate of 1/2 in^3/min. How fast is the height of water increasing when h=2in?





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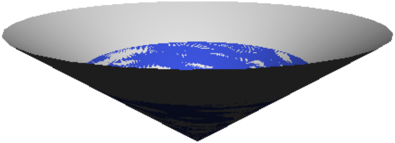

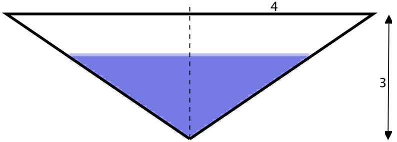

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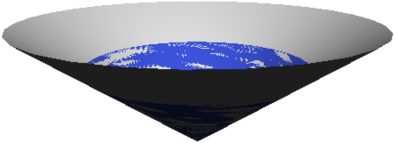

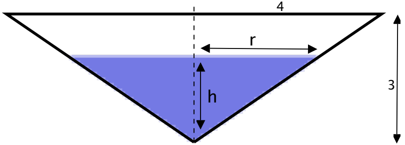

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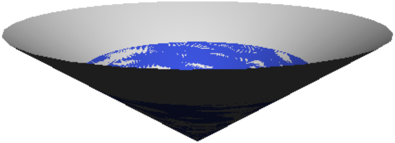

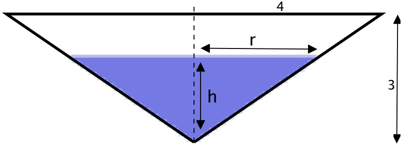

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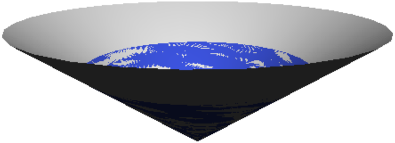

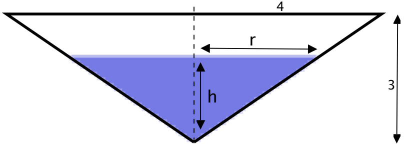

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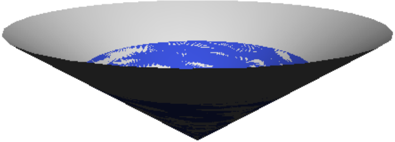

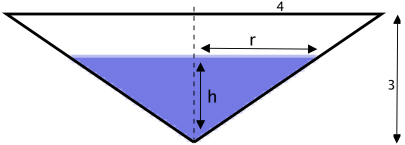

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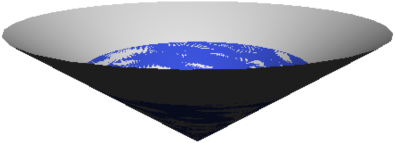

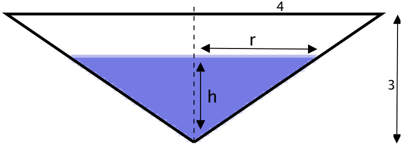

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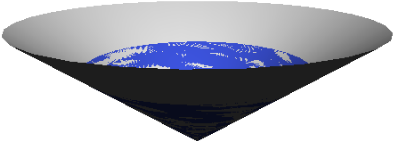

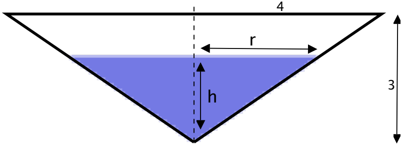

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So $\boxed{\left.\frac{dh}{dt}\right|_{h=2} = \frac{9}{128\pi}}$

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4. Solve for the rate you want.

One more example: (from extra problems)

10. A boat is pulled into a dock by a rope attached to the bow of the boat a passing through a pulley on the dock that is 1 m higher than the bow of the boat. If the rope is pulled in at a rate of 1 m/s how fast is the boat approaching the dock when it is 8 m from the dock?

On your own: (from extra problems)

5. A balloon which always remains spherical is being inflated by pumping in 900 cubic centimeters of gas per second. Find the rate at which the radius of the balloon is increasing when the radius is 15cm.

11. Gravel is being dumped from a conveyor belt at a rate of 30 ft^3/min and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 10 ft high?

9. A lighthouse is on a small island 3 km away from the nearest point P on a straight shoreline and its light turns four revolutions per minute. How fast is the beam of light moving along the shoreline when it is 1 km from P? [hint: 4rpm means that some angle is changing at $4 * 2\pi$ radians per minute]