# Exponential and Logarithm Functions

# The Basics

If $n$ and $m$ are positive integers...

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**Some identities:**

**Examples:**

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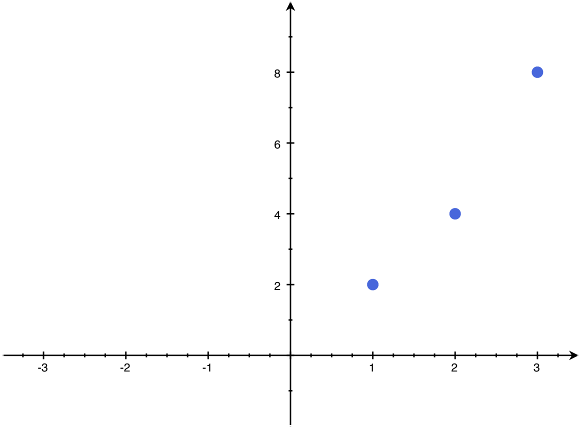
Example: $8^{5/3} = \left(\sqrt[3]{8}\right)^5 = 2^5 = 32$ or $8^{5/3} = \sqrt[3]{8^5} = \sqrt[3]{32{,}768} = 32$

# What is $a^x$ for all $x$?

If $a > 1$:

(e.g. $a = 2$)

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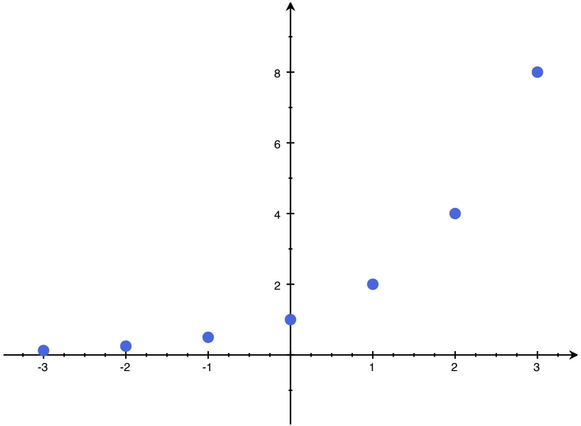


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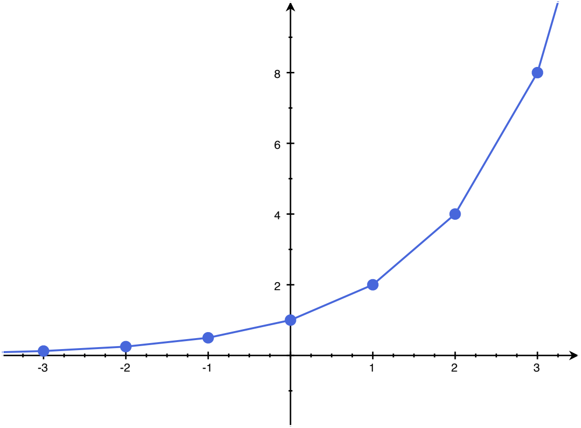


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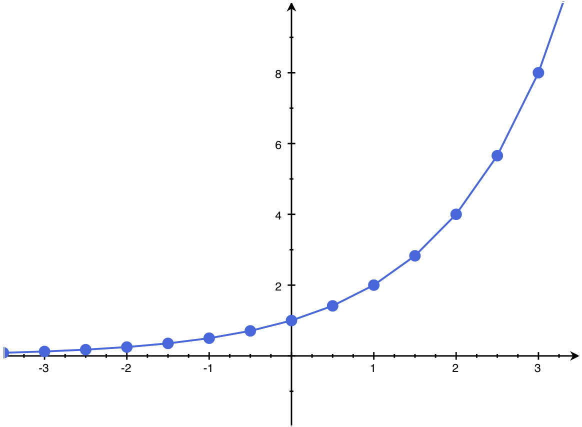


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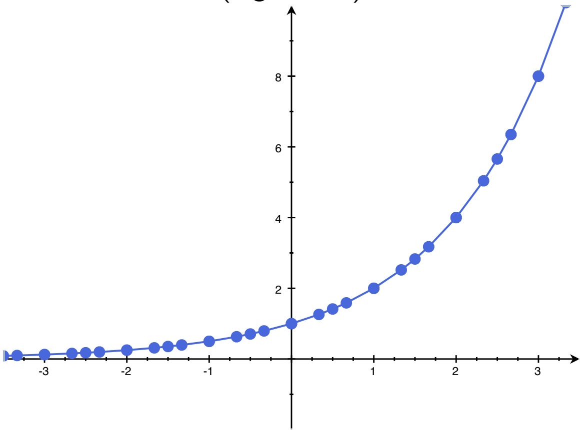


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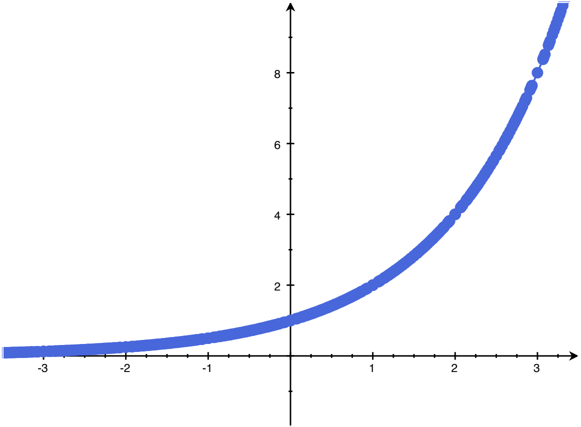


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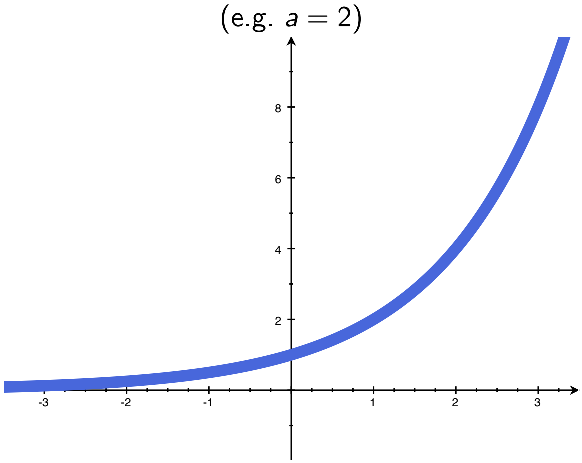


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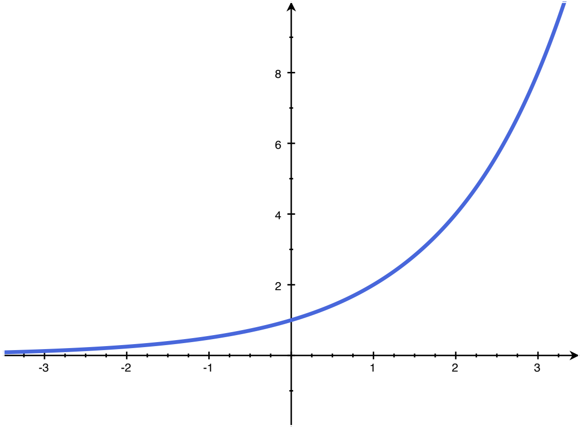


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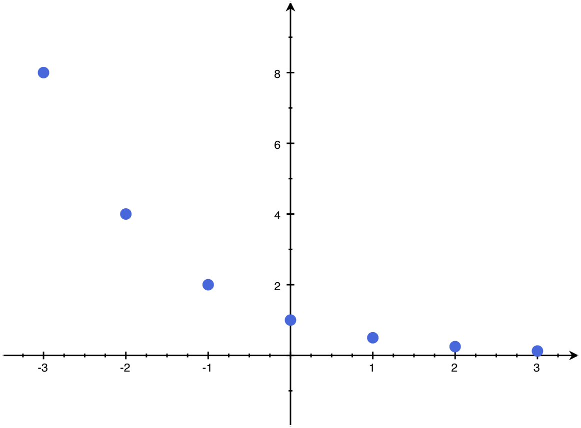


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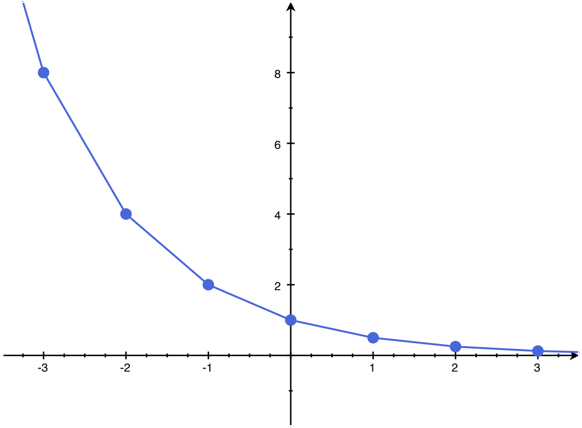


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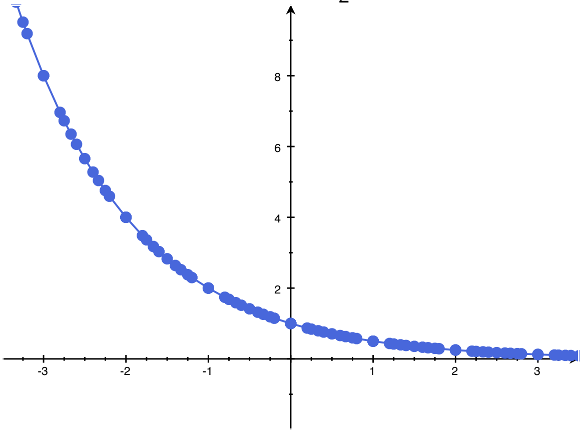


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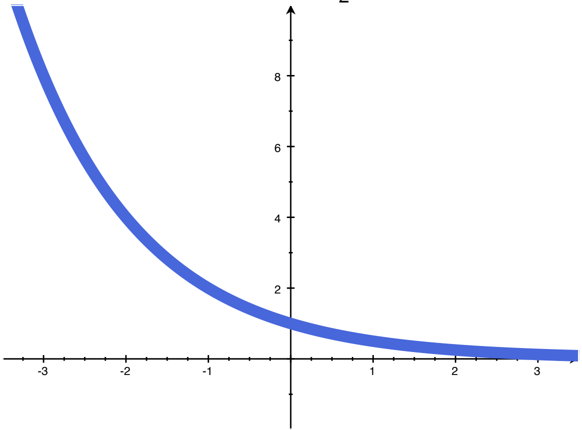


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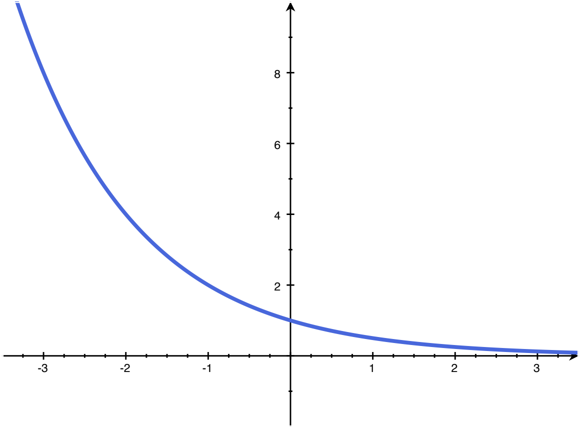


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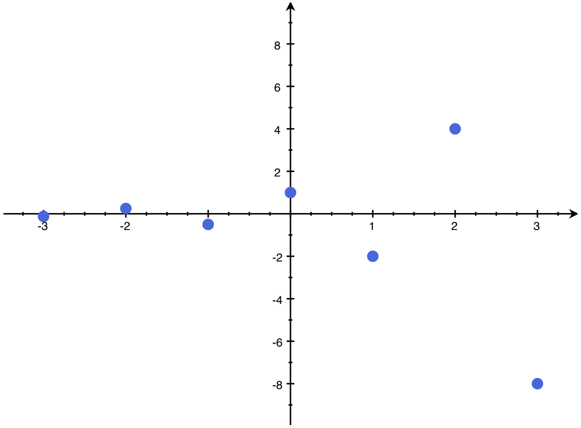


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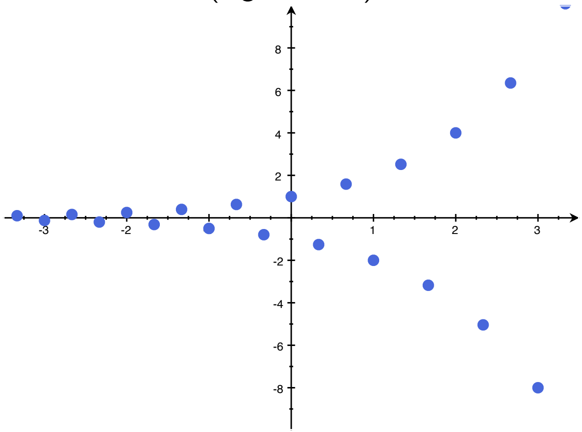


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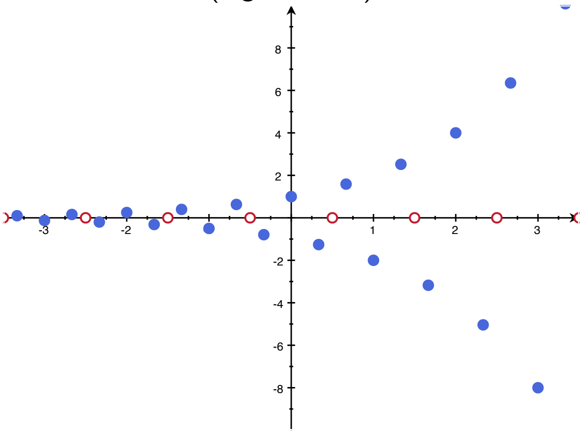


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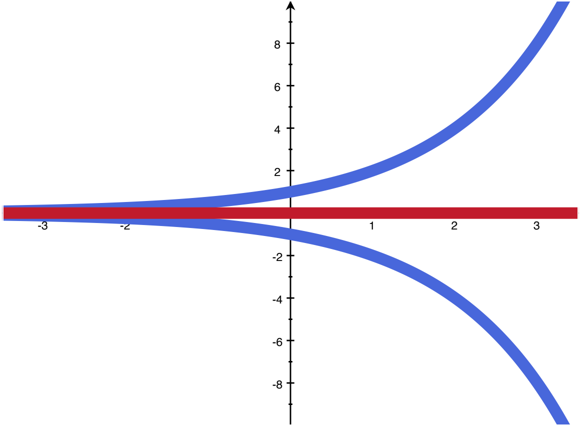


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OH NO!

# The function $a^x$:

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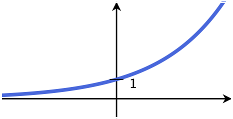


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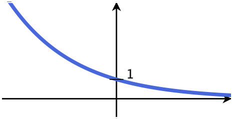


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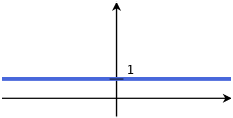


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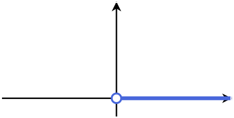


D: $(0, \infty)$, R: $\{0\}$

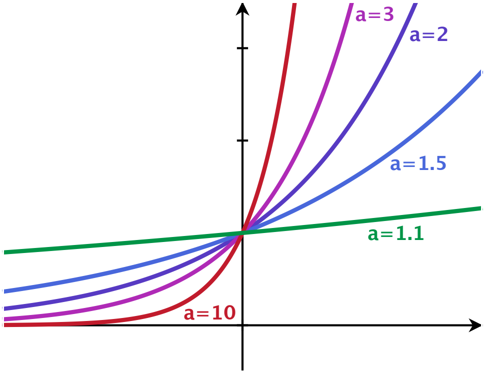
**Properties:**

$a^b * a^c = a^{b+c}$ $\quad (a^b)^c = a^{b*c}$ $\quad a^{-x} = 1/a^x$ $\quad a^c * b^c = (ab)^c$

# Our favorite exponential function:

Look at how the function is increasing through the point $(0, 1)$:

$y = a^x$ :



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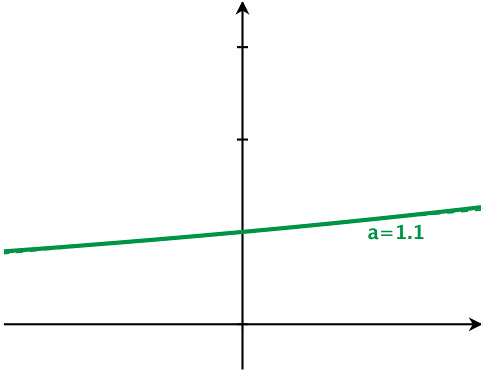

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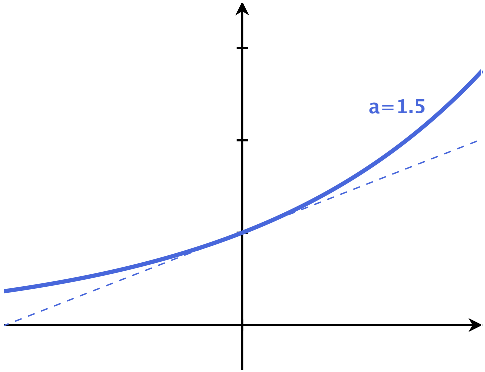

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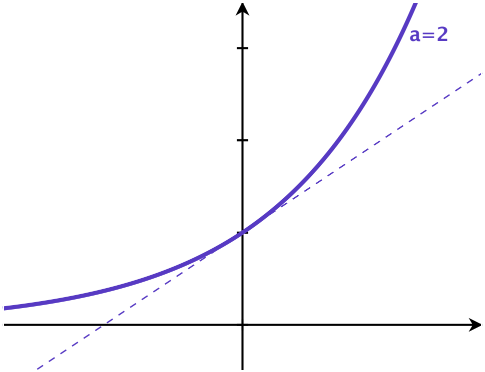

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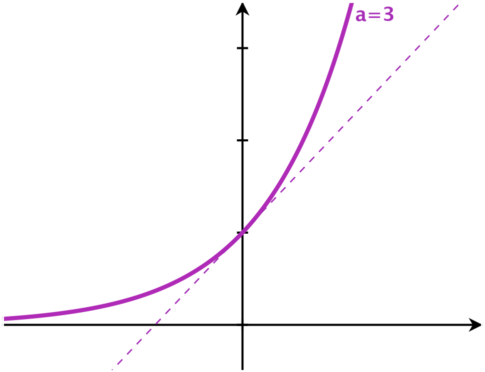

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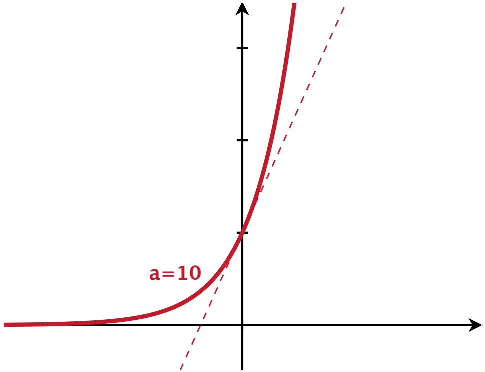

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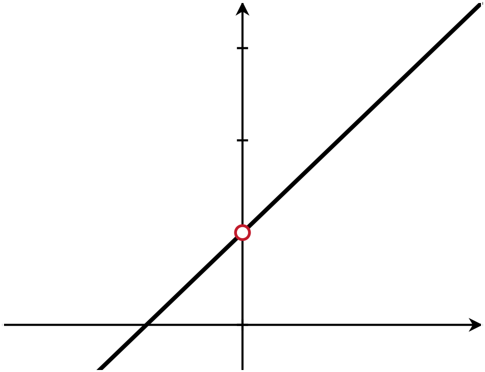

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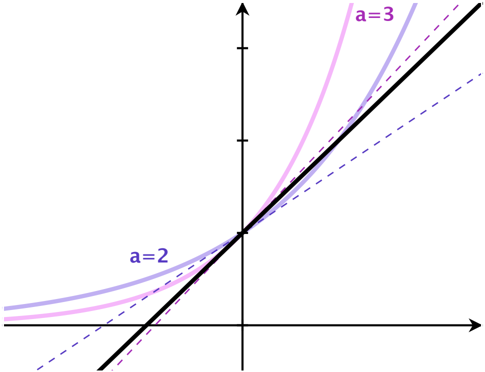


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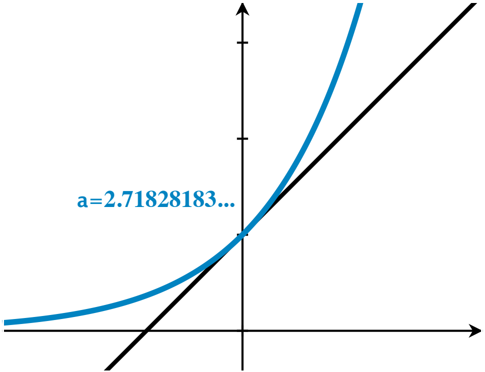


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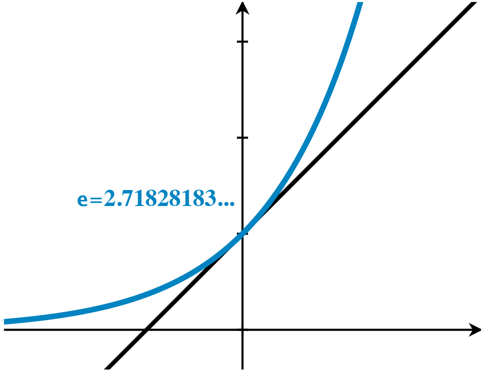


**Q:** Is there an exponential function whose slope at (0,1) is 1?

**A:** $e^x$ is the exponential function whose slope at (0,1) is 1.

($e = 2.71828183\ldots$ is to calculus as $\pi = 3.14159265\ldots$ is to geometry)

# Logarithms

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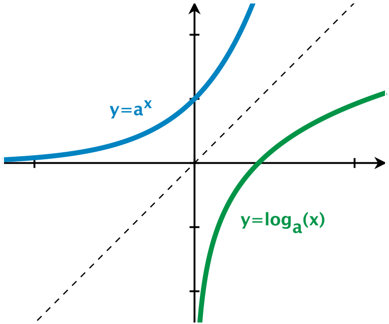
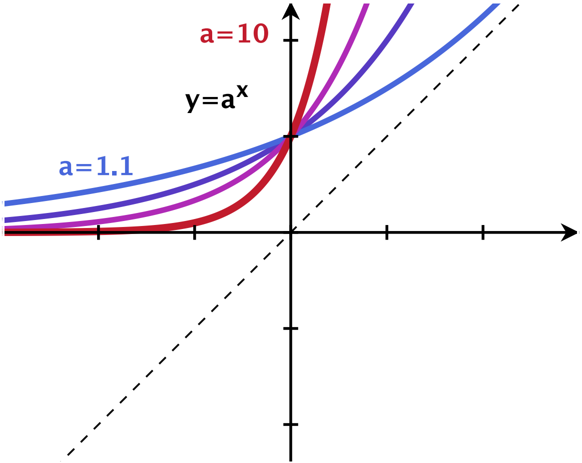


# Properties of Logarithms



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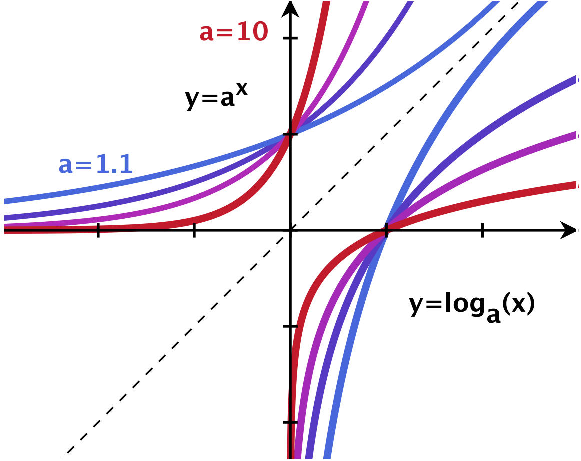

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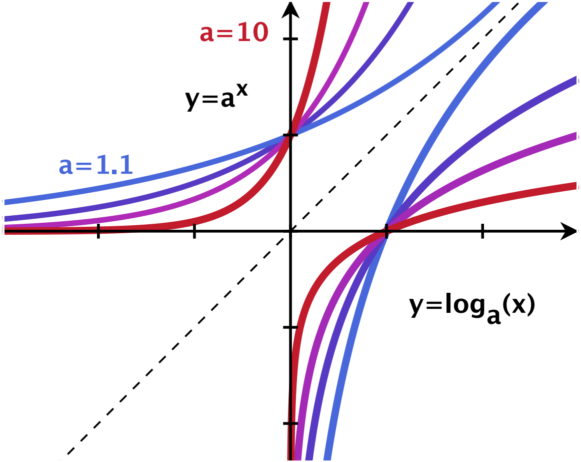


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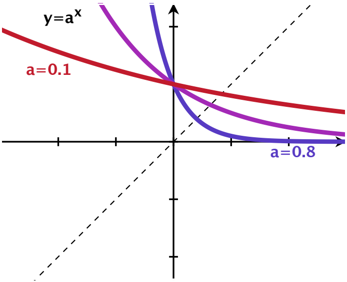


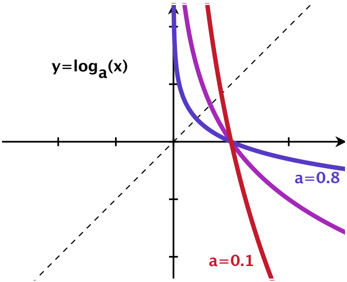


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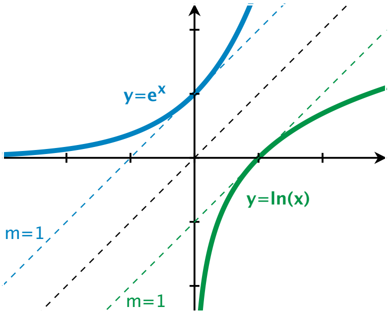
Lastly: $\dfrac{\log_a(b)}{\log_a(c)} = \log_c(b)$

# Favorite logarithmic function

Remember: $y = e^x$ is the function whose slope through the point (0,1) is 1.
The *inverse* to $y = e^x$ is the *natural log*:

$$\ln(x) = \log_e(x)$$



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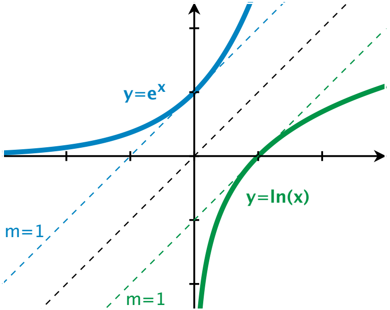


We will often use the facts that $e^{\ln(x)} = x$ $_{\text{(for } x>0)}$ and $\ln(e^x) = x$ $_{\text{(for all } x)}$

# Two super useful facts:

Explain why:

(1) $\log_a(b) = \ln(b)/\ln(a)$

(2) $a^b = e^{b\ln(a)}$ [hint: start by rewriting $b\ln(a)$, and use the fact that $e^{\ln(x)} = x$]

# Examples:

(1) Condense the logarithmic expressions

$$\frac{1}{2}\ln(x)+3\ln(x+1) \qquad 2\ln(x+5)-\ln(x) \qquad \frac{1}{3}(\log_3(x)-\log_3(x+1))$$

(2) Solve the following expressions for $x$:

$$e^{-x^2} = e^{-3x-4} \qquad 3(2^x) = 24$$

$$2(e^{3x-5}) - 5 = 11 \qquad \ln(3x+1) - \ln(5) = \ln(2x)$$