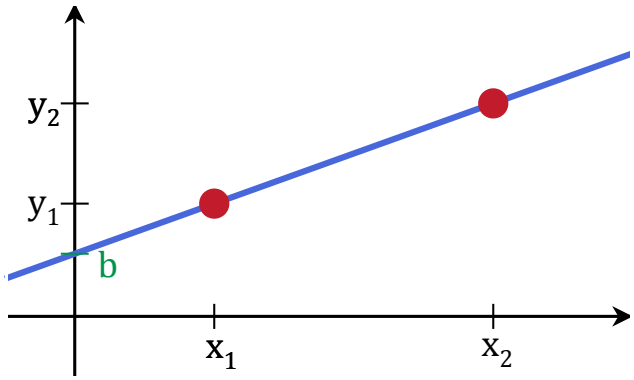
# Functions and their graphs

## Simplest functions: Lines!




Two points define a line!

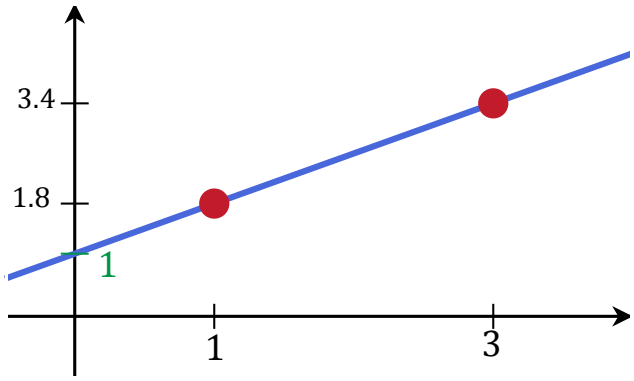
Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ (rise/run)

Point-slope form: $y - y_1 = m(x - x_1)$ (good for writing down lines)

Slope-intercept form: $y = mx + b$ (good for graphing)

General form: $Ax + By + C = 0$ (accounts for $\infty$ slope)

# Simplest functions: Lines!




Example:

Slope: $m = \dfrac{3.4 - 1.8}{3 - 1} = 0.8$ (rise/run)

Point-slope form: $y - 1.8 = 0.8 * (x - 1)$ (good for writing down lines)

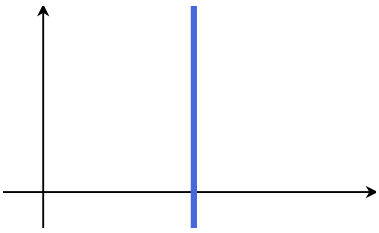
Slope-intercept form: $y = 0.8 * x + 1$ (good for graphing)

General form: $0.8 * x + y - 1 = 0$ (accounts for $\infty$ slope)

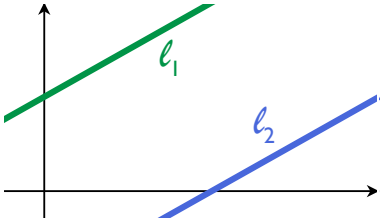
# Lines: Special cases

Constant functions
$(m = 0)$

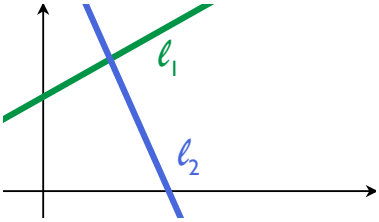
Vertical lines
$(m = \infty)$



Parallel lines
$(m_1 = m_2)$




Perpendicular lines
$(m_1 = -1/m_2)$



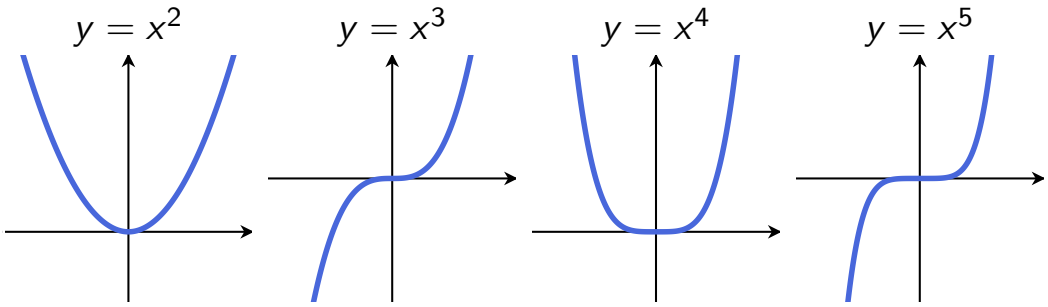
## Other good functions to know: polynomials.

$$y = a_0 + a_1 x + \cdots + a_n x^n$$

($n$ is the *degree*)

The basics (know these graphs!)

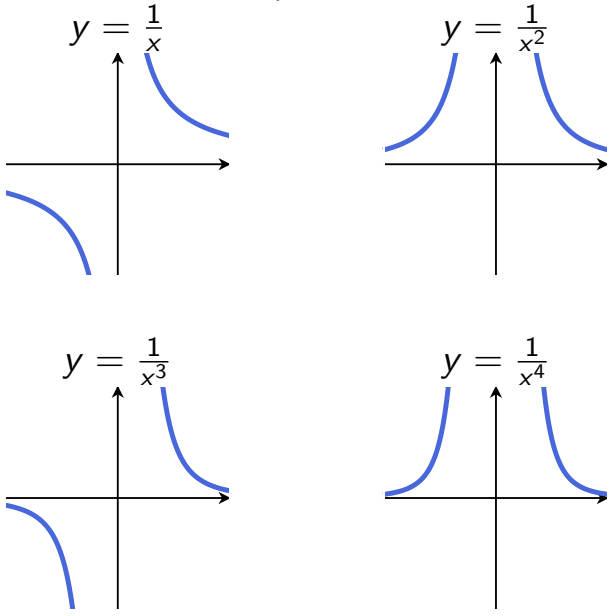
$n = 0$: constants $\quad$ $n = 1$: lines $\quad$ $n = 2$: parabolas




## Other good functions to know: rationals.

$$y = \frac{a_0 + a_1 x + \cdots + a_n x^n}{b_0 + b_1 x + \cdots + b_m x^m}$$

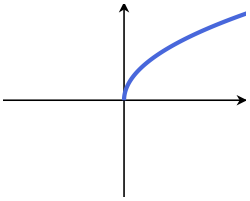
The basics (know these graphs!)



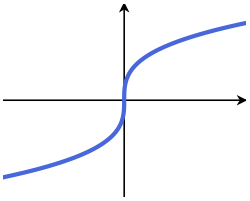
Other powers: $y = x^a$.

The basics (know these graphs!)

$y = x^{1/2} = \sqrt{x}$

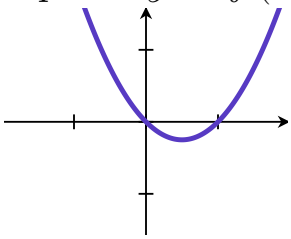


$y = x^{1/3} = \sqrt[3]{x}$

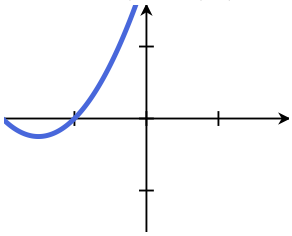


# New functions from old

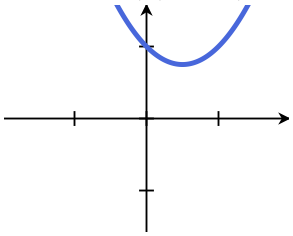
Graph of $y = f(x)$:



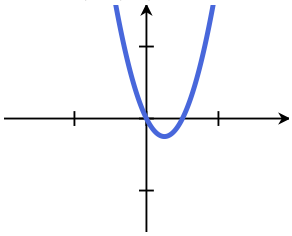
Graph of $y = f(x+2)$ (left shift) :



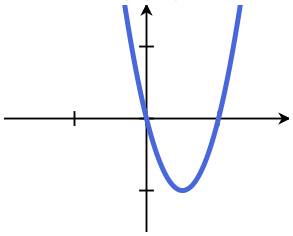
Graph of $y = f(x) + 1$ (up shift):



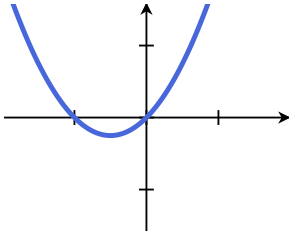
Graph of $y = f(2x)$ (horizontal squeeze) :



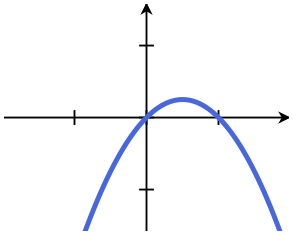
Graph of $y = 4 * f(x)$ (vertical dialation):



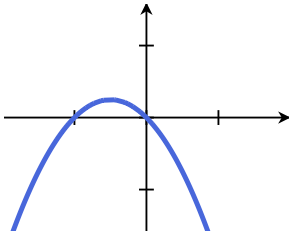
Graph of $y = f(-x)$ (vertical reflection) :



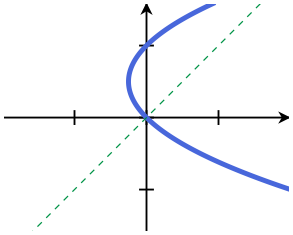
Graph of $y = -f(x)$ (horizontal reflection):



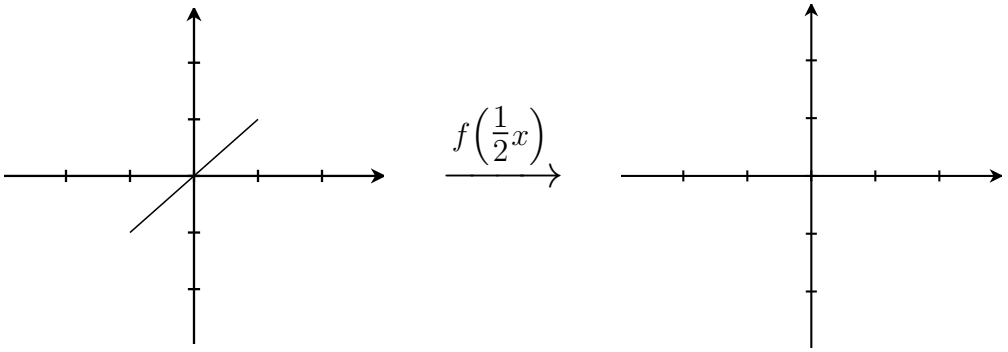
Graph of $-y = f(-x)$ (180° rotation) :

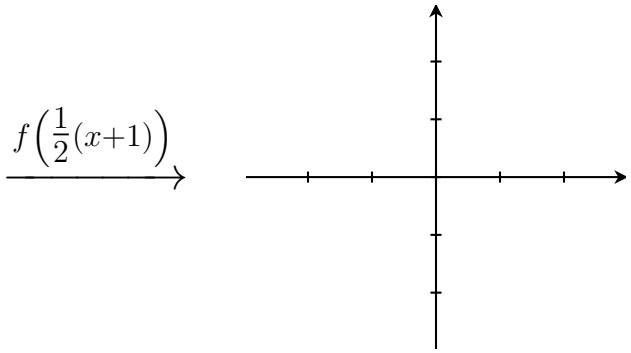


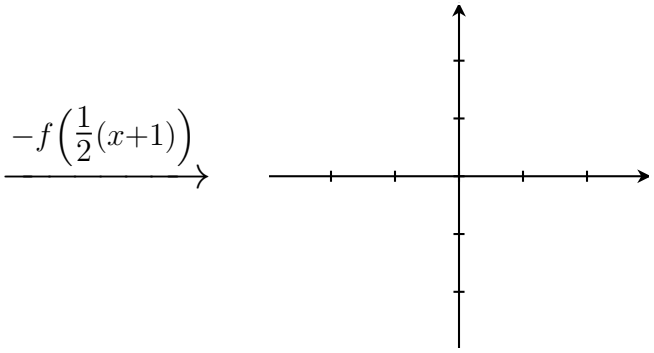
Graph of $x = f(y)$ (flip over $y = x$):

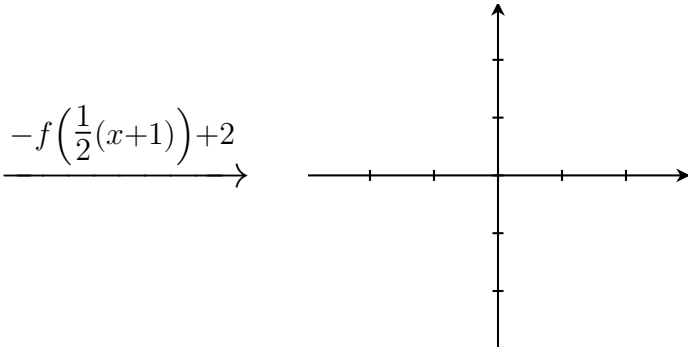


Ex: Transform the graph of $f(x)$ into the graph of $-f\left(\frac{1}{2}(x+1)\right)+2$:




The *domain* of a function $f$ is the set of $x$ over which $f(x)$ is defined.

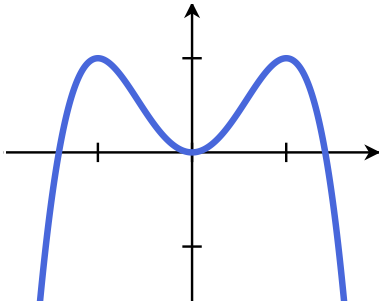
The *range* of a function $f$ is the set of $y$ which satisfy $y = f(x)$ for some $x$.

# Symmetries

A function $f(x)$ is *even* if it satisfies

$$f(-x) = f(x)$$

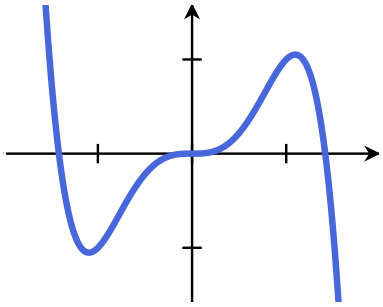
ex: $f(x) = 2x^2 - x^4$



A function $f(x)$ is *odd* if it satisfies

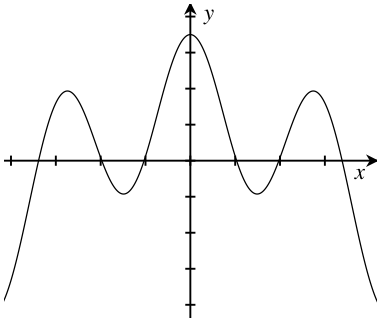
$$f(-x) = -f(x)$$

ex: $f(x) = 2x^3 - x^5$

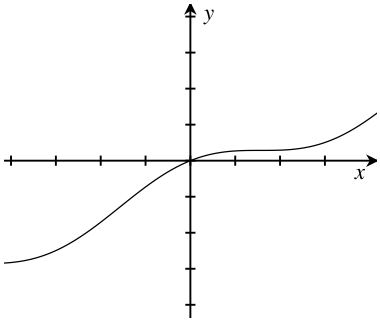


## Examples: Even, odd, or neither?

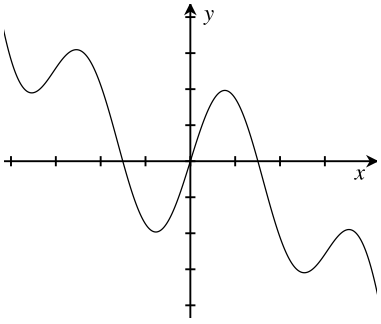
(a)




(b)




(c)

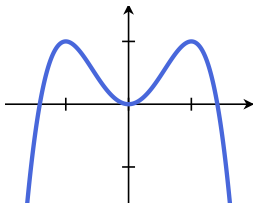



(d) $f(x) = \dfrac{x^3 + x}{x + \frac{1}{x}}$

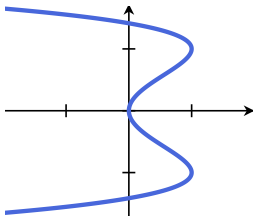
(for this one:
actually plug in $-x$
and see what happens
algebraically)

A graph is a graph of a *function* if for every $x$ in its domain, there is exactly one $y$ on the graph which is mapped to by that $x$:

Function:

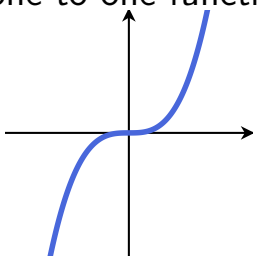


Not a function:



A function is additionally *one-to-one* if for every $y$, there is at most one $x$ which maps to that $y$.

A one-to-one function:



## Inverse functions

Let $f$ be a one-to-one function.
If $g$ is a function satisfying

$$f(g(x)) = g(f(x)) = x$$

then $g$ is the *inverse function* of $f$. Write $g(x) = f^{-1}(x)$.

**Example:** If $f(x) = x^3$, then $f^{-1}(x) = \sqrt[3]{x}$

To calculate $f^{-1}(x)$, set $f(y) = x$ and solve for $y$. Then $y = f^{-1}(x)$.

**Example:** If $f(x) = \frac{2x}{x-1}$,

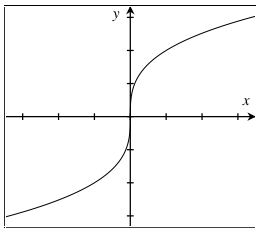
$$\text{solve } x = \frac{2y}{y-1} \text{ for } y \text{ to get } y = \frac{x}{x-2}.$$

So $f^{-1}(x) = \frac{x}{x-2}$.

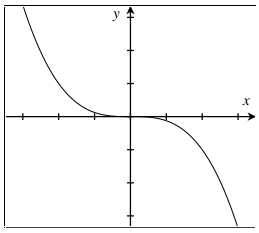
To get the graph of $f^{-1}(x)$, flip the graph of $f(x)$ over the line $y = x$.

Pair up graphs with their inverses:

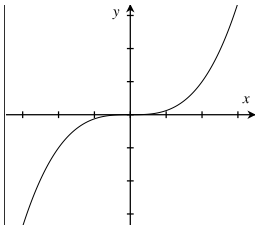
A




B




C

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