HW15 10 pts. 1

3.3 #3
$$\frac{dx}{dt} = e^{x}\sin t$$
$$\Downarrow$$
$$e^{-x}\,dx = \sin t\,dt$$
$$\Downarrow$$
$$\int e^{-x}\,dx = \int \sin t\,dt$$
$$\Downarrow$$
$$\cancel{-}e^{-x} = \cancel{-}\cos t + C$$
$$\Downarrow$$
$e^{-x} = \cos t + C$ (C is an arbitrary constant, so sign doesn't matter)
$$\Downarrow$$
$$-x = \ln(\cos t + c)$$
$$\Downarrow$$
$x = -\ln(\cos t + c)$. $(\cos t + c > 0)$

#4
$$\frac{dy}{dx} = 6y + 4 \Longrightarrow \boxed{y \equiv -\frac{2}{3}}$$ ← This is also one of the soln's
$$\Downarrow$$
$\frac{1}{6y+4}\,dy = dx$ (If $y \neq -\frac{2}{3}$)

$$\int \frac{1}{6y+4}\,dy = \int 1 \cdot dx$$

$$\Downarrow$$

$$\frac{1}{6}\ln|6y+4| = x + c$$

$$\Downarrow$$

$$\ln|6y+4| = 6x + c$$ (scale of c doesn't matter)

$$\Downarrow$$

$$|6y+4| = e^{6x+c} = ce^{6x}$$

$$\Downarrow$$

If $y > -\frac{2}{3}$, $6y+4 = ce^{6x}$

$$\Downarrow$$

$$y = \boxed{\frac{ce^{6x}-4}{6}}$$

since c's sign doesn't matter, process is the same.

If $y < -\frac{2}{3}$, $6y+4 = -ce^{6x}$

* This Answer also includes situat'n where $y \equiv -\frac{2}{3}$. $(c=0)$

#7 $$dy = -9 \cdot \frac{1}{\sec(4x)}\,dx$$

$$= -9\cos(4x)\,dx$$

$$\int 1 \cdot dy = -9 \int \cos(4x)\, dx$$

$$\Downarrow$$

$$y = -\frac{9}{4} \cdot \sin(4x) + c$$

3.4 #2 (1) $y'(2) = y(2) = 2.1.$

$$y(2.5) \approx y(2) + 0.5 \times y'(2)$$

$$= 2.1 + 0.5 \times 2.1$$

$$= 3.15$$

$$y'(2.5) = y(2.5) = 3.15$$

$$y(3) \approx y(2.5) + 0.5 \times y'(2.5)$$

$$= 3.15 \times 1.5$$

$$= 4.725$$

(2) $\frac{dy}{dx} = y \Rightarrow \frac{1}{y} dy = 1 \cdot dx \Rightarrow \ln|y| = x + c$

$$\Downarrow$$

$$y = ce^x$$

$$2.1 = ce^2 \Rightarrow c = \frac{2.1}{e^2}$$

$$y(3) = \frac{2.1}{e^2} \cdot e^3 = 2.1e = 5.708.$$

#3 (1) use ~~formula~~ $y(x_0+h) = y(x_0) + h y'(x_0)$.

in the same way as in #2.

$y(5) \approx$ ~~0.7854~~ ~~7.00~~ -0.7009.

(2) $\frac{dy}{dx} = \cos x - \sin x$

$\Downarrow$

$1 \cdot dy = (\cos x - \sin x)\,dx$

$\Downarrow$ Integrate both sides

$y = \sin x + \cos x + C$

$1 = 0 + 1 + C \Rightarrow C = 0$

so $y = \sin x + \cos x$

$\Downarrow$

$y(5) = \sin 5 + \cos 5$

$= -0.6753$