

### Notes on Chapter 2

§ 2.1. Transversal theory has not yet acquired a standardized terminology, and expressions such as 'system of distinct representatives', 'common transversal', 'common system of representatives' and so on are used in varying senses by different authors. Our own choice has primarily been guided by the need to avoid confusion between sets and families. It should be noted that the terminology adopted in this book is not identical with that of the survey article by Mirsky & Perfect (1).

The independence of the  $2^n - 1$  statements which constitute condition  $\mathcal{H}$  was noted by R. Rado (3).

§ 2.2. Theorem 2.2.1 was proved by P. Hall (1) in 1935. It is implicit in the earlier literature and has, for this reason, been sometimes associated with the names of Dénes König and E. Egerváry. Thus, as we have seen above, it is an almost immediate consequence of König's theorem 1.7.1 on bipartite graphs. However, it is precisely Hall's formulation that has turned out to be the master key which has unlocked many closed doors.

Hall's original proof of Theorem 2.2.1 was comparatively difficult. Other proofs of this or of closely related results have since been given by a number of writers, among them W. Maak (1), Marshall Hall Jr. (2), Weyl (1), Everett & Whaples (1), Halmos & Vaughan (1), D. Gale (2, 143-6), and R. Rado (11). The first of the proofs of Hall's theorem offered above is that of Halmos & Vaughan. The very transparent argument can be adapted for coping with more general situations (cf. for example Exs. 3.3.4 and 6.2.3). The second proof, due to Rado, is equally simple and the process of 'reduction' introduced here can, when combined with Zorn's lemma, be used to establish the transfinite form of Hall's theorem (see § 4.2). It can also be used to discuss 'independent' transversals (see § 6.2). D. J. A. Welsh (7) exploited the method of reduction to obtain generalizations of Hall's theorem.

An efficient algorithm for identifying a transversal of a given family or else demonstrating that no transversal exists has been devised by M. Hall Jr. (3).

A discussion of the relation of Hall's theorem to boolean algebra will be found in Hammer & Rudeanu (1, 252-257).

§ 2.3. The observation that in transversal theory there is a rather obvious duality between sets and elements must have been made by almost everyone attempting research in this field. The idea is implicit in the work of Edmonds & Fulkerson (1) and is developed explicitly by Mirsky & Perfect (2). The terminology employed in the present account is taken largely from the latter paper.

For much interesting work on deltoids, see J. S. Pym (1).