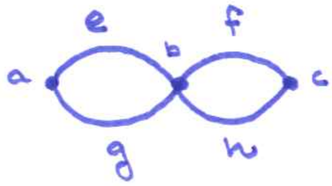
Quiz 1, Math 38, Spring 2012

Decide which of the following are possible. For those which are possible, give an example. For those which are not possible, explain (or prove) why not. In either case, justify your answers.

(1) G is bipartite and contains a 3-clique.

Not possible. Bipartite graphs cannot have odd cycles.

(2) G has a closed trail which is not a cycle (if so, give the trail).



Walk:

a, e, b, f, c, h, b, g, a

(3) G is a simple digraph whose underlying graph is not simple.

underlying graph:

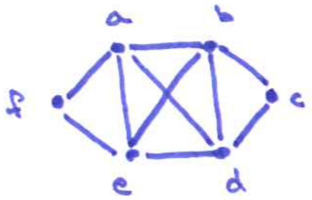
(multiple edges!)

(4) G is 3-regular with $n(G) = 5$.

Not possible since $\sum_i d(v_i)$ must be even.

(3-regular w/ $n(G)=5$ would yield $\sum_i d(v_i) = 3 \cdot 5 = 15$)

(5) G is Eulerian and has K_4 as an induced subgraph.



Graph is Eulerian <u>iff</u> Graph is even.

$G - \{f, c\} \cong K_4$

(6) G is simple and has degree sequence $5, 4, 4, 2, 2, 1$.

$5, 4, 4, 2, 2, 1$ is graphic <u>iff</u>

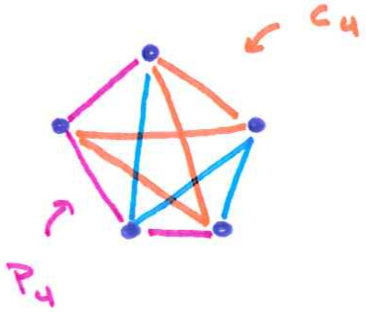
$3, 3, 1, 1, 0$ is graphic, which is <u>iff</u>

$2, 0, 0, 0$ is graphic, which it isn't.

$\Rightarrow$ not possible.

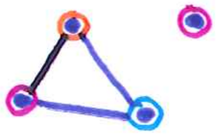
(7) G has P_4 and C_4 as subgraphs, but not as induced subgraphs.

K_5:



All induced subgraphs are complete!

(8) G is disconnected and 3-colorable, but is not 2-colorable.



3-cliques imply not 2-colorable, but here's a 3-coloring.

No path between the isolated vertex and any other vertex.

(9) G has exactly two vertices of odd degree, each lying in separate components of G.

Not possible: each component is itself a graph, and must have $\sum_i d(v_i)$ even.

(the induced subgraph by deleting all vertices not in a component has the same degrees as before for each of the vertices in the component)