Math 36: Mathematical Modeling **Quiz 1**
Fall 2017

Name: Answer Key

Instructions: This is a 20–minute quiz consisting of true/false and short answer questions. No justification is required for True/False or computational questions. Each question is worth ten points.

1. **T/F questions:**

   (a) __T__ In the standard logistic equation, the change in P_n is proportional to $(M - P_n)P_n$.
   Answer: The logistic equation is $P_{n+1} = P_n + k(M - P_n)P_n$ so the change is $P_{n+1} - P_n = k(M - P_n)P_n$

   (b) __T__ In the equation $P_{n+1} = P_n + .2P_n$, the growth rate of P_n is exponential for $P_0 > 0$.
   Answer: Simplifying to $P_{n+1} = 1.2P_n$ shows the exponential growth

   (c) __F__ The Chebyshev criterion is concerned with the size of the smallest residual.

   Answer: The criterion is minimizing the size of the *largest* residual

   (d) __F__ Given data (x_i, y_i) and model $(x_i, f(x_i))$, the L^1 error is always less than the L^2 error.

   Answer: Consider a single data point where the residual is .1.

   (e) __T__ The equation $P_{n+1} = .3P_n + 4$ has a stable equilibrium point.

   Answer: The ratio (.3) is less than 1 so the equilibrium point is stable.

2. **Consider the difference equation $P_{n+1} = aP_n + b$.**

   (a) If $P_0 = 2$, $a = 3$, and $b = -1$, what is P_2? $P_2 =$ _____14_____.

   Answer:
   $$P_1 = 3(2) - 5 = 5 \qquad P_2 = 3 * 5 - 1 = 14$$

   (b) If $b = 7$, write a value for a that guarantees that an unstable equilibrium: $a =$ _____2_____.

   Answer: Any value for a that is greater than 1 will be unstable.
   (c) What is the equilibrium value for your equation from part (b)? _____-7_____

   Answer: Given $P_{n+1} = aP_n + b$ the equilibrium point is given by $\dfrac{b}{1-a} \rightarrow \dfrac{7}{1-2} = -7$.

3. **A group of islanders is divided into two groups. Group A of people who are aware of sea otters and group U who are unaware. Every day, each person on the island talks to each other person exactly once. In each conversation between a person in group A and a person in group U, there is a 23% chance that the person in group U becomes aware of of sea otters and joins group A.**

(a) Assuming that no islanders die and no new islanders are born, write a system of difference equations describing how the numbers of people in A and U change over time.
Answer: Each day there are $A_n U_n$ interactions between people in different groups and 23% of those interactions cause a person to move from U to A. As a system, this looks like:

$$A_{n+1} = A_n + .23 A_n U_n$$
$$U_{n+1} = U_n - .23 A_n U_n$$

(b) If $A_0 = 2$ and $U_0 = 98$, what is the eventual distribution between the groups as $n \to \infty$?

$A_n \to$ _____100_____ $U_n \to$ _____0_____

Answer: Since the population isn't changing, eventually everyone is aware.

4. **Consider the data and model in the table below for this problem.**

x_i	1.1	2.2	3.5	4.6	6.7
y_i	1.5	3.6	17	32	140
$f(x_i)$	.5	6.6	15	32	139

(a) Compute the residuals for the data and model above.

$1.5 - .5 = 1$ $3.6 - 6.6 = -3$ $17 - 15 = 2$ $32 - 32 = 0$ $140 - 139 = 1$

(b) Using the residuals you computed in part (a) compute the three error measures:

$L^\infty =$ _____3_____ $L^1 =$ ___7 or $\frac{7}{5}$___ $L^2 =$ ___15 or $\sqrt{15}$ or 3 or $\frac{\sqrt{15}}{3}$___

5.

(a) For the equation $P_n = n^2 + 2$ what is ΔP_n?

Answer: $P_{n+1} - P_n = (n+1)^2 + 2 - (n^2 + 2) = 2n + 1$

(b) What is the most interesting thing you have learned so far in this course?