MATH 36 DAILY ASSIGNMENT #8 (LINEAR ALGEBRA APPROACH)

We want to find the steady state of the following system of equations:

$$\begin{bmatrix} a_{n+1} \\ b_{n+1} \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} a_n \\ b_n \end{bmatrix} + \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

Setting $a_{n+1} = a_n$ and $b_{n+1} = b_n$ we can do some algebra:

$$\begin{aligned}
\begin{bmatrix} a_n \\ b_n \end{bmatrix} &= \begin{bmatrix} 4 & 4 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} a_n \\ b_n \end{bmatrix} + \begin{bmatrix} -2 \\ 1 \end{bmatrix} \\
\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 4 \\ 2 & -2 \end{bmatrix} \right) \begin{bmatrix} a_n \\ b_n \end{bmatrix} &= \begin{bmatrix} -2 \\ 1 \end{bmatrix} \\
\begin{bmatrix} -3 & -4 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} a_n \\ b_n \end{bmatrix} &= \begin{bmatrix} -2 \\ 1 \end{bmatrix} \\
\begin{bmatrix} a_n \\ b_n \end{bmatrix} &= \frac{1}{17} \begin{bmatrix} -3 & -4 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \end{bmatrix} \\
\begin{bmatrix} a_n \\ b_n \end{bmatrix} &= \begin{bmatrix} 2/17 \\ 7/17 \end{bmatrix}
\end{aligned}$$

This gives us our steady state and suggests how we might evaluate the stability. Instead of looking directly at $X_n = \begin{bmatrix} a_n \\ b_n \end{bmatrix}$ we instead consider the equivalent equation:

$$\begin{bmatrix} a_{n+1} \\ b_{n+1} \end{bmatrix} - \begin{bmatrix} 2/17 \\ 7/17 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 2 & -2 \end{bmatrix} \left(\begin{bmatrix} a_n \\ b_n \end{bmatrix} - \begin{bmatrix} 2/17 \\ 7/17 \end{bmatrix} \right)$$

Although this is a simple modification this makes it clear why the eigenvalues of $\begin{bmatrix} 4 & 4 \\ 2 & -2 \end{bmatrix}$ control the stability since the left hand side of the equation is the difference between the current state and the equilibrium point. Since the eigenvalues of the matrix are $1 \pm \sqrt{17}$ which are both greater than 1 in magnitude it must be the case that even a small deviation from the equilibrium point will lead to exponential divergence. On the other hand, if the eigenvalues had been smaller than 1 then the equilibrium point would have been stable.

The attached MatLab code demonstrates this behavior. I adjusted the plot so that the y-axis is logarithmic so you can see the divergence. Running it with

```
>> m36hw8(2/17,7/17);
```

shows the expected steady state, while running it with

```
>> m36hw8(2/17+.000001,7/17+.000001);
```

shows the divergence and the instability of the system (next page).

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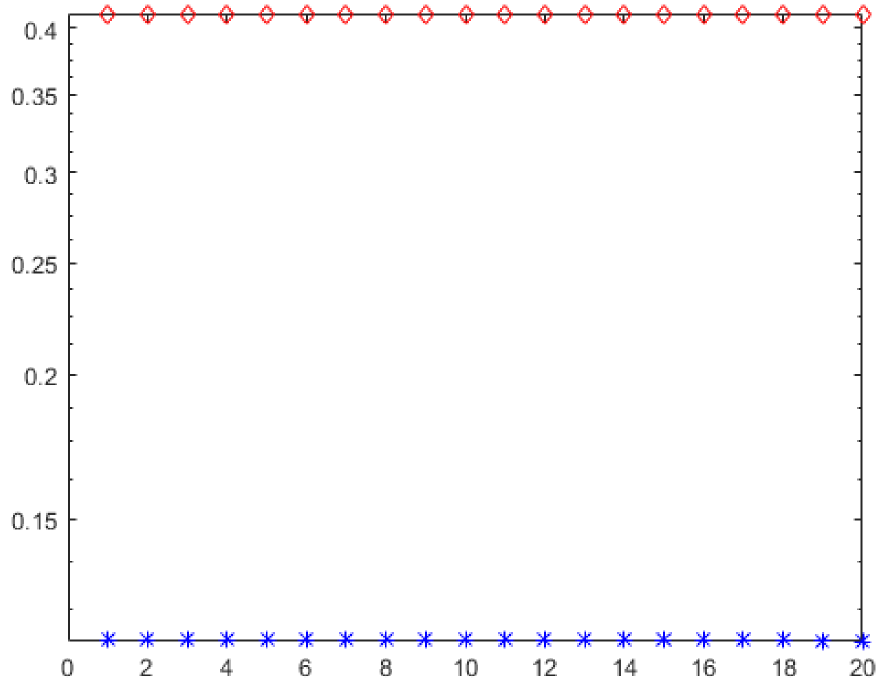
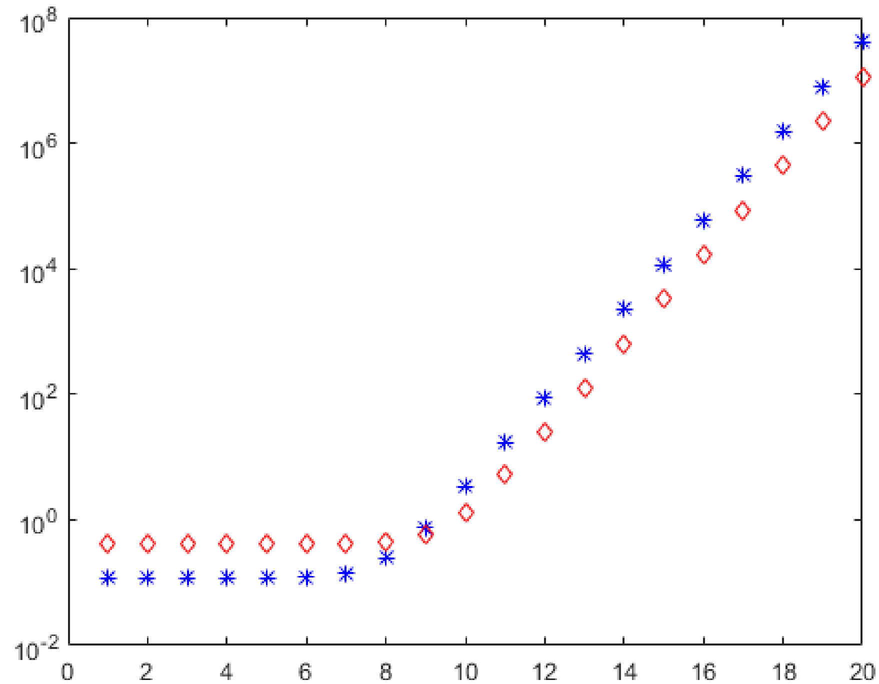
Date: September 28, 2017.



0.4
0.35
0.3
0.25
0.2
0.15
0
2
4
6
8
10
12
14
16
18
20



10^8
10^6
10^4
10^2
10^0
10^{-2}
0
2
4
6
8
10
12
14
16
18
20