MATH35 EXAM

Due date: Friday's class (Feb 10th 11:30am).

You can use textbook and notes for reference, but no discussion!

Problem 1: Let f be a continuous function on $\mathbb{R}$ and $\{x_k\}_{k=1}^{\infty}$ is a Cauchy sequence. Prove that $\{f(x_k)\}_{k=1}^{\infty}$ is a Cauchy sequence as well.

Problem 2: Suppose that $\{x_k\}_{k=1}^{\infty}$ is an unbounded sequence but doesn't converge to infinity. Show that there are two subsequences $\{x_{p_n}\}$ and $\{x_{q_n}\}$ such that $\{x_{p_n}\}$ is convergent and $\lim_{n\to+\infty} x_{q_n} = \infty$ (that is, either $\lim_{n\to+\infty} x_{q_n} = +\infty$ or $\lim_{n\to+\infty} x_{q_n} = -\infty$).

Problem 3: Suppose that $f(x)$ is a function on $(0, +\infty)$, satisfying that $f(2x) = f(x)$. If $\lim_{x\to\infty} f(x) = A$, prove that

$$f(x) \equiv A, \qquad x \in (0, +\infty).$$

Problem 4: Suppose that $f(x)$ is a continuous function on (a, b) such that

$$\lim_{x\to a^+} f(x) = A, \qquad \lim_{x\to b^-} f(x) = B.$$

Prove that $f(x)$ must be uniformly continuous.

Problem 5:

- Suppose that $\lim_{x\to 0} f(x^3) = A$, where A is a constant number. Prove that $\lim_{x\to 0} f(x) = A$.

- If $\lim_{x \to 0} f(x^2) = A$, can we still conclude that $\lim_{x \to 0} f(x) = A$? If YES, please prove it. Otherwise, please give one counter example.