Math 35
Winter 2014
Homework Assigned Wednesday, January 8

As usual, this homework is due at the beginning of class next Monday, January 13.

Problem: Prove that the natural numbers are closed under addition; that is, for all natural number m and n, the sum $m + n$ is also a natural number.

Use the principle of mathematical induction, the fact that the natural numbers are closed under adding 1, and any axioms or theorems about ordered fields you wish.

Suggestion: Use induction on n. That is, let m be any natural number. Use the principle of mathematical induction to prove the statement

> For every natural number n, the sum $m + n$ is a natural number.

On the next page is an example of a proof by mathematical induction. (I will have given this proof in class, if anyone asked.)

Proposition: For every natural number n, we have $0 < n$.

Proof: By induction on n

Base Case: We must show $0 < 1$. This was proven in the homework assigned on January 6.

Inductive Step: Assume as inductive hypothesis that n is a natural number such that $0 < n$. We must show $0 < n + 1$.

By the axioms for an ordered field, whenever $0 < a$ and $0 < b$, then $0 < a + b$. Because $0 < n$ by inductive hypothesis, and $0 < 1$ by the base case, therefore, $0 < n + 1$. This is what we needed to show. $\square$