Homework Assignment 8

Due Friday May 25

1. Solve by separation of variables:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} + \gamma^2 u, \qquad 0 \le x \le a \qquad 0 < t$$

$$u(0,t) = 0, \qquad u(a,t) = 0 \qquad 0 < t$$

$$u(x,0) = h \qquad u_t(x,o) = 0 \qquad 0 < x < a$$

2.

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} - \frac{1}{T}F(x,t)$$

Change variables to

$$w = x + ct, \quad z = x - ct, \quad u(x,t) = v(w,z), \quad f(w,z) = F(x,t)$$

The equation become

$$\frac{\partial^2 v}{\partial w \partial z} = -\frac{1}{T}f(z,w)$$

Show that the general solution to this equation is

$$v(z,w) = -\frac{1}{4T}\int\int f(z,w)dwdz + \phi(w) + \psi(z)$$

Find the general solution to the equation in term of x and t,
if $F(x,t) = T\cos(t)$.

3. From the book page 251 ex 1,3,6

4. Solve the potential equation $\nabla u = 0$ in the disk $0 < r < c$ if the boundary condition:

   (a) $u(c,\theta) = |\theta|$, $-\pi < \theta < \pi$

   (b) $u(c,\theta) = \theta$, $-\pi < \theta < \pi$