Homework Assignment 7

Due Friday May 18

1. The pressure of the air in an organ pipe satisfy the equation:

$$\frac{\partial^2 p}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}, \qquad 0 \le x \le a \qquad 0 < t$$

With the boundary conditions (p_0 is atmospheric pressure)

(a) $p(0,t) = p_0$, $p(a,t) = p_0$ if the pipe is open, or

(b) $p(0,t) = p_0$, $\frac{\partial p}{\partial t}(a,t) = 0$ if the pipe is closed at $x = a$

(a) Find the eigenvalues and eigenfunctions associated with the wave equation for each of these sets of boundary conditions.

(b) Find the lowest frequency of vibration of the air in the organ pipes for each of these sets of boundary conditions.

2. If a string vibrate in a medium that resists the motion, the problem for the displacement of the string is:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} + k\frac{\partial u}{\partial t}, \qquad 0 \le x \le a \qquad 0 < t$$
$$u(0,t) = 0 \qquad u(a,t) = 0 \qquad 0 < t$$

plus initial conditions. Find the eigenvalues and eigenfunctions, and product solutions for this problem (assume that k is small and positive)

3. Solve the problem:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{\alpha^2}\frac{\partial u}{\partial t}, \qquad 0 \le x \le \infty \qquad 0 < t$$

$$u(0,t) = \begin{cases} T_1 & 0 < x < b \\ 0 & x < b \end{cases} \qquad u(x,t) \to 0 \text{ as } x \to \infty \; 0 < t$$

$$u(x,0) = 0, \qquad 0 < x < \infty$$