Homework Assignment 3

Due Monday April 16

1. Lets $S_n(t) = \sum_{k=0}^{n} c_k(f) e^{i\pi kt}$. where c_k is the complex Fourier coefficient.
   prove that
   $$S_n(t) = \int_{-\pi}^{\pi} D_n(t-u) f(u) du$$
   Where
   $$D_n(x) = \frac{\sin(\frac{(2n+1)x}{2})}{2\sin(\frac{x}{2})}$$

2. Find the Fourier series of the function $f(x) = \cos(px)$ where p is not an integer. and show that:
   $$\pi \cot(p\pi) = \frac{1}{p} + 2\sum_{n=1}^{\infty} \frac{p}{p^2 - n^2}$$
   $$\frac{\pi}{\sin(p\pi)} = \frac{1}{p} + 2\sum_{n=1}^{\infty} \frac{(-1)^n p}{p^2 - n^2}$$

3. Show that
   $$c_n(fg) = \sum_{p=-\infty}^{\infty} c_{p-n}(f) c_p(g)$$

4. The Fourier series of $f(x) = \frac{\sin(x)}{x}$, $-\pi \leq x \leq \pi$ converge at every point (why?) to what value the series converge at $x = 0$ and at $x = \pi$?