Math 31 Take Home Midterm
Due July 25, 2018

Instructions: This is an open notes, open book exam. You may use your notes or homework assignments from class, as well as Pinter 2nd edition. You may not use the internet or any other textbooks, nor may you confer with classmates. If you use any unauthorized aid, this is an honor code violation. This exam is due at the beginning of class on July 25.

1. Let G be a group of permutations on a set X, and let $a \in X$. Then

$$\text{Stab}(a) = \{\alpha \in G : \alpha(a) = a\}$$

   is called the **stabilizer** of a in G. Prove that $\text{Stab}(a)$ is a subgroup of G.

2. Prove that if a subgroup has index 2, then it is a normal subgroup.

3. Recall that the centralizer of an element $a \in G$ is defined by

$$C(a) = \{g \in G : ga = ag\}.$$

   Find the centralizer of (123) in S_3.

4. If $a, g \in G$ where G is a group, prove that $C(a) \cong C(gag^{-1})$.

5. Prove that $\langle \{2^m 3^n : m, n \in \mathbb{Z}\}, \cdot \rangle$ is not cyclic.

6. Let G be a group of order 25. Prove that either G is cyclic, or $g^5 = e$ for all $g \in G$.

7. Let H be a subgroup of G, and let $a \in G$ be a fixed element.
   (a) Prove that aHa^{-1} is a subgroup of G of the same order as H.
   (b) Let $|H| = n$ and let H be the unique subgroup of G of order n. Prove that $H \trianglelefteq G$.