Math 31 Homework 7
Due August 17, 2018

1. Let R be a ring.
   (a) An element $a \in R$ is called **nilpotent** if $a^n = 0$ for some positive integer n. Show that 0 is the only nilpotent element in an integral domain.
   (b) An element $a \in R$ is called **idempotent** if $a^2 = a$. Prove that 0 and 1 are the only idempotent elements in an integral domain.
2. Let $i = \sqrt{-1}$. Is $\mathbb{Z}_5[i] = \{a + bi : a, b \in \mathbb{Z}_5\}$ an integral domain? If so, prove that it is. If not, give a counterexample.
3. Chapter 20, part A
   (a) Exercise 1
   (b) Exercise 2
4. Let R and S be rings and let $\phi : R \to S$ be a ring homomorphism.
   (a) Prove $\mathrm{Ker}(\phi)$ is an ideal of R.
   (b) Prove the first isomorphism theorem for rings: $R/\mathrm{Ker}(\phi) \cong \mathrm{Im}(\phi)$.
5. (a) Chapter 18, exercise D6.
   (b) Chapter 18, exercise F7.
   (c) Chapter 20, exercise E4.
   (d) Chapter 20, exercise F2.
6. Chapter 24
   (a) Exercise A2
   (b) Exercise C2
7. Chapter 24, exercise E2.