

Math 31 Homework 2

Due July 6, 2018

Do not use Chapter 10 of the textbook when solving these problems!

1. Prove that in the Cayley table of a group, every element appears exactly once in each row and each column.
2. Prove that a group G is abelian if and only if it has the property that if $ab = ca$, then $b = c$ for all $a, b, c \in G$.
3. Let x belong to a group. If $x^2 \neq e$ and $x^6 = e$, prove that $x^4 \neq e$ and $x^5 \neq e$. What does this say about the order of x ?
4. Suppose $|a| = n$.
 - (a) For any $m \geq n$, prove that $a^m = a^i$ for some $i \in \{0, \dots, n-1\}$.
 - (b) Prove that for all $r, s \in \{0, \dots, n-1\}$, if $r \neq s$ then $a^r \neq a^s$.
5. In chapter 5, section A, do problems:
 - (a) problem 5
 - (b) problem 6
6. Chapter 5 exercise D1
7. Let G be a group and let $a \in G$. Prove that $\langle a \rangle = \langle a^{-1} \rangle$.
8. In chapter 11, section D, do problems:
 - (a) problem 1
 - (b) problem 2