Math 31 Homework 1

Due June 29, 2018

1. Are the following binary operations? Why or why not?

   (a) $a * b = a\ln(b)$ on the set of positive real numbers ($\{x \in \mathbb{R} : x > 0\}$).

   (b) $a * b = |a - b|$ on the set of non-negative integers ($\{n \in \mathbb{Z} : n \geq 0\}$).

2. Apply the directions from Chapter 2, exercise B to the operation $x * y = x + y - xy$.

3. Do the following Cayley tables represent groups? Why or why not? If so, is the group represented abelian? Why or why not? (If a Cayley table has both an identity and inverses, then I have made it associative - you do not need to check this. Ordinarily, you may not assume that an operation is associative, but we do not know any good ways aside from brute force to check associativity in Cayley tables, and I don't want to make you do that!)

(a)

	a	b	c	d
a	a	b	c	d
b	c	b	a	d
c	b	a	d	c
d	d	c	b	a

(b)

	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	c	d	b
d	d	b	a	c

(c)

	a	b	c	d
a	c	a	d	b
b	a	b	c	d
c	d	c	b	a
d	b	d	a	c

4. Write out the Cayley table(s) for all group(s) with two elements (call them a and b).

   (a) For each that you found, prove that it is a group.

   (b) Why are the other possible Cayley tables not groups? (you do not need to prove that each individual table is not a group, but do explain why you did not include them in your list).

   (c) We discussed $\mathbb{Z}_n$ in class. Make a Cayley table for $\mathbb{Z}_2$. Is this Cayley table the same as one that you found above? Which one? Why are they the same?

   (d) What conclusion can you make about the number of distinct groups with two elements?

5. We talked about D_4 in class. For D_3 (symmetries of an equilateral triangle), do the following:

   (a) Describe the group elements.

   (b) Write out the Cayley table.

   (c) Select two triples and show that they are associative (this is not enough to prove that the whole operation is associative).

   (d) Prove that D_3 is a group (you may assume associativity).

   (e) Based on this problem and what we did with D_4 in class, how many elements do you expect D_n, the group of symmetries on an n-gon, to have? What do you expect these elements to look like?