Quiz 1

Composition laws, groups and subgroups

1. Consider the composition law[1] defined on $\mathbb{Z}$ by

$$p * q = p - 2q.$$

Is it associative? If so, prove it. Otherwise, supply a counterexample.

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[1]You do **not** need to verify that it is a composition law.

2. The composition law defined on $\Gamma = \{(x, y) \in \mathbb{R} \times \mathbb{R} \, , \, y \neq 0\}$ by

$$(a, b) \boxtimes (c, d) = (ad + bc, bd)$$

is associative[2]. Is $(\Gamma, \boxtimes)$ a group?

3. Let G be a group with neutral element e and a, b elements in G satisfying

$$a^{-1}ba^{-1} = b^{-1}ab^{-1}.$$

Solve simultaneously the equations $ax^2 = b$ and $x^3 = e$.

4. Let $H = \left\{ \dfrac{p}{2^n} \ , \ p \in \mathbb{Z} \ , \ p \neq 0 \ , \ n \in \mathbb{N} \right\}$. Is H a subgroup of $(\mathbb{Q}^\times, \times)$?

Bonus. Name an Italian mathematician from the Renaissance.