Major Facts about Polynomial Rings

Fact 1. **If** R is an integral domain, **then** $R[x]$ is an integral domain as well.

Fact 2. **(Division Algorithm for $F[x]$)** Let F be a **field** and let $f(x), g(x) \in F[x]$, such that $g(x) \neq 0$. **Then** there exist unique polynomials $q(x), r(x) \in F[x]$ such that $f(x) = g(x)q(x) + r(x)$ and either $r(x) = 0$ or $\deg r(x) < \deg g(x)$.

Corollary 2.1. **(The Remainder Theorem)** Let F be a field, $a \in F$, and $f \in F[x]$.
Then $f(a)$ is the **remainder** in the division of $f(x)$ by $x - a$.

Corollary 2.2. **(The Factor Theorem)** Let F be a field, $a \in F$, and $f \in F[x]$.
Then a is a zero of $f(x)$ **if and only if** $x - a$ is a factor of $f(x)$.

Corollary 2.3. Let F be a field and $f \in F[x]$ a polynomial of degree n.
Then $f(x)$ has at most n zeros **counting multiplicity**.