

EXAMPLES OF GROUPS

Group	Description of Elements	General Form of Elements	Operation	Identity	Inverse	Abelian?
\mathbb{Z}	integers	k	addition	0	$-k$	Yes
\mathbb{Q}^+	positive rational numbers	m/n with $m, n > 0$	multiplication	1	n/m	Yes
\mathbb{Z}_n	integers modulo n	k	addition mod n	0	$n - k$	Yes
\mathbb{R}^*	nonzero real numbers	x	multiplication	1	$1/x$	Yes
$U(n)$	integers $< n$ that are relatively prime with n	k such that $\gcd(k, n) = 1$	multiplication mod n	1	solution to $kx \bmod n = 1$	Yes
$M_{m \times n}(\mathbb{Z})$ $M_{m \times n}(\mathbb{R})$ $M_{m \times n}(\mathbb{Z}_p)$	$m \times n$ matrices A with entries from $\mathbb{Z}, \mathbb{R}, \mathbb{Z}_p$	for $m = 2, n = 3$: $A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$	matrix addition	zero matrix	$-A$	Yes
$GL(n, \mathbb{R})$ $GL(n, \mathbb{Q})$ $GL(n, \mathbb{C})$	$n \times n$ matrices A with entries from $\mathbb{R}, \mathbb{Q}, \mathbb{C}$ and $\det A \neq 0$	for $n = 2$: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, with $ad - bc \neq 0$	matrix multiplication	identity matrix I_n	for $n = 2$: $\frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$	No
$SL(n, \mathbb{Z})$ $SL(n, \mathbb{R})$ $SL(n, \mathbb{Q})$	$n \times n$ matrices A with entries from $\mathbb{Z}, \mathbb{R}, \mathbb{Q}$ and $\det A = 1$	for $n = 2$: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, with $ad - bc = 1$	matrix multiplication	identity matrix I_n	for $n = 2$: $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$	No
V vector space	vectors	\mathbf{v}	vector addition	$\mathbf{0}$	$-\mathbf{v}$	Yes
D_n	symmetries of an n -gon	R_α (rotations) and L (reflections)	composition	R_0	$R_{360-\alpha}$ and L	No