

# MAJOR FACTS ABOUT NORMAL SUBGROUPS AND FACTOR GROUPS

- FACT 1. (**Normal Subgroup Test**) Let  $G$  be a group and  $H \leq G$  its subgroup. **Then**  $H$  is normal in  $G$  **if and only if**  $xHx^{-1} \subset H$  for all  $x \in G$ .
- FACT 2. (**Factor Groups**) Let  $G$  be a group and  $H \triangleleft G$  its normal subgroup. **Then** the set  $G/H = \{aH | a \in G\}$  is a group under the operation  $(aH)(bH) = abH$ .
- FACT 3. (**Factor Group by the Center**) Let  $G$  be a group and  $Z(G)$  its center. **If**  $G/Z(G)$  is cyclic, **then**  $G$  is Abelian.
- FACT 4. Let  $G$  be a group. **Then**  $G/Z(G) \approx \text{Inn}(G)$ .
- FACT 5. (**Cauchy's Theorem for Abelian Groups**) Let  $G$  be a finite Abelian group and let  $p$  be prime such that  $p \mid |G|$ . **Then**  $G$  has an element of order  $p$ .