Major Facts about Cosets

Theorem 1. **(Properties of Cosets)** Let G be a group, H its subgroup, and $a, b \in G$. **Then**

a. $a \in aH$;

b. $aH = H$ **if and only if** $a \in H$;

c. $aH = bH$ **or** $aH \cap bH = \varnothing$;

d. $aH = bH$ **if and only if** $a^{-1}b \in H$;

e. $|aH| = |bH|$;

f. $aH = Ha$ **if and only if** $H = aHa^{-1}$;

g. aH is a subgroup of G **if and only if** $a \in H$.

Theorem 2. **(Lagrange's Theorem)** Let G be a finite group and H its subgroup. **Then** $|H|$ divides $|G|$. **Moreover,** the number of distinct left (or right) cosets of H in G is $|G|/|H|$.

Corollary 2.1. $|G{:}H| = |G|/|H|$, where $|G{:}H|$ is the index of H in G.

Corollary 2.2. $|a|$ divides $|G|$ for all $a \in G$.

Corollary 2.3. A group of prime order is cyclic.

Corollary 2.4. $a^{|G|} = e$ for all $a \in G$.

Corollary 2.5. **(Fermat's Little Theorem)**

$a^p \equiv a \bmod p$ for every integer a and every prime p.

Theorem 3. **(Classification of Groups of order $2p$)** Let $p > 2$ be a prime number and G be a group of order $2p$. **Then** either $G \approx \mathbb{Z}_{2p}$ or $G \approx D_p$.