Math 31: Abstract Algebra
Fall 2018 - Quiz 2

Date: 10/23/18

Test your knowledge

True false questions *(1 points each)*

1. Let $p_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 4 & 9 & 2 & 3 & 8 & 1 & 6 & 5 \end{pmatrix}$ be a permutation in $(S_9, \circ)$.
   Then $p_1 = (17) \circ (24) \circ (68) \circ (395)$. ◯ True ◯ False

2. Let $p_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 9 & 5 & 3 & 1 & 2 & 4 & 8 & 6 \end{pmatrix}$ be a permutation in $(S_9, \circ)$.
   Then $p_2 = (43517) \circ (296)$. ◯ True ◯ False

3. Let $p_3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 9 & 8 & 7 & 4 & 3 & 6 & 5 & 1 & 2 \end{pmatrix}$ be a permutation in $(S_9, \circ)$.
   Then $p_3^{37} = p_3$. ◯ True ◯ False

4. For any two cycles $b, c \in (S_n, \circ)$ we have that $c \circ b = b \circ c$. ◯ True ◯ False

5. For two cycles $a, b \in (S_n, \circ)$, which have no number in common, we always have that $a \circ b = b \circ a$.
   ◯ True ◯ False

6. The set $S_{\mathbb{R}} = \{f : \mathbb{R} \to \mathbb{R}, f \text{ bijective }\}$ is a subgroup of $(\mathcal{F}(\mathbb{R}), +)$. ◯ True ◯ False

7. In $\mathcal{F}(\mathbb{R})$ let $\sim$ be the relation defined by: $f \sim g \Leftrightarrow f(3) = g(3)$. Then $\sim$ is an equivalence relation. ◯ True ◯ False

8. Let $(G, \cdot)$ be a group and H a subgroup. For a fixed $a \in G$ the function
   $f : aH \to Ha, ah \mapsto f(ah) := ha$ is a bijective function. ◯ True ◯ False

9. Let $(G, \cdot)$ be a group and H a subgroup. Then for any $a \in G$ we have $aHa^{-1} = H$.
   ◯ True ◯ False

10. Let $(G, \cdot)$ be a group and H a subgroup. Then if $a \in Hb$ then $Ha = Hb$ and if $a \notin Hb$ then $Ha \neq Hb$. ◯ True ◯ False

Long answer questions

question 1 *(6 points)* Write down the cosets of the subgroup $\langle \frac{1}{3} \rangle$ generated by $\frac{1}{3}$.

a) For $\langle \frac{1}{3} \rangle \leq (\mathbb{R}^*, \cdot)$.

b) For $\langle \frac{1}{3} \rangle \leq (\mathbb{R}, +)$.

question 2 *(4 points)* Let $(G, \cdot)$ be a group and $H \leq G$ a subgroup and $N \lhd G$ be a normal subgroup. Show that

$$H \cdot N = \{h \cdot n, h \in H, n \in N\} \text{ is a subgroup of } G.$$