Math 31: Abstract Algebra
Fall 2018 - Quiz 1

Date: 10/02/18

Test your knowledge

True false questions *(1 point each)*

1. $+_4$ is an operation on the set $\mathbb{Z}_2 = \{0, 1\}$. ◯ True ◯ False

2. Let $*$ be an operation on a set A. If $(A, *)$ has a neutral element e, then e is unique. ◯ True ◯ False

3. Let $(G, \cdot)$ be a group and $a, b \in G$. Then $(ab)^2 = a^2b^2$. ◯ True ◯ False

4. Let $(G, \cdot)$ be a group and H and K subgroups of G. Then $H \cup K$ is a subgroup of G. ◯ True ◯ False

5. The set $H = \{f : \mathbb{R} \to \mathbb{R} \mid f(x) \geq 0 \text{ for all } x \in \mathbb{R}\}$ is a subgroup of $(\mathcal{F}(\mathbb{R}), +)$. ◯ True ◯ False

6. Let $(G, \cdot)$ be a group, $a, b \in G$ fixed and $f : G \to G, x \mapsto f(x) = axb$. Then f is bijective. ◯ True ◯ False

7. Let $(G, \cdot)$ be a group. $S \subset G$, such that $\#S = n$ and $\langle S \rangle = G$. Then G has only finitely many elements. ◯ True ◯ False

8. If G and H are groups, such that $\#G = n$ and $\#H = m$. Then $\#(G \times H) = n + m$. ◯ True ◯ False

9. $(\mathcal{F}(\mathbb{R}), \cdot)$ is a group with neutral element $1 : \mathbb{R} \to \mathbb{R}, x \mapsto 1(x) = 1$. ◯ True ◯ False

10. $(\mathbb{Q}, +)$ is isomorphic to $(\mathbb{Z}, +)$. **Hint:** If $F : \mathbb{Q} \to \mathbb{Z}$ is an isomorphism. If $F(q) = 1$, what is $F(\frac{q}{2})$? ◯ True ◯ False

Long answer questions

question 1 *(5 points)* Let $G = \{e, a, b, c\}$ be a set of four elements, where e denotes the neutral element. Using an operation table, find all possible groups with four elements, where each element is its own inverse.

question 2 *(5 points)* Let $(G, \cdot)$ be a group and $H = \langle\{a, b\}\rangle$ be the subgroup generated by the elements a and b, which satisfy the equations

$$a^2 = e \quad , \quad b^3 = e \quad , \quad ab = ba.$$

a) Show that H is an abelian group.

b) How many different elements can H contain at most?