

Math 31: Abstract Algebra
Fall 2018 - Homework 1

Return date: **Thursday 09/20/18**

keywords: *operations - examples and properties*

Instructions: Write your answers neatly and clearly on straight-edged paper, use complete sentences and label any diagrams. Please show your work; no credit is given for solutions without work or justification.

exercise 1. (*6 points*) Which of the following rules are operations on the indicated set? Justify your answer in each case.

- a) $a * b = a \cdot b$, on the set $\{n \in \mathbb{Z} : n \leq 0\}$.
- b) $(a, c) * (b, d) = (a + b, c + d)$, on the set $\mathbb{R} \times \mathbb{R} \setminus \{(0, 0)\}$.
- c) $a * b = |a^2 - b|$, on the set $\{n \in \mathbb{Z} : n \geq 0\}$.

exercise 2. (*6 points*) Each of the following is an operation $*$ on \mathbb{R} .

- a) $x * y = |x + y|$.
- b) $x * y = x + 2y + 1$.
- c) $x * y = \max\{x + 1, y + 1\}$.

Explain whether or not

- i) the operation is commutative,
- ii) the operation is associative,
- iii) \mathbb{R} has an identity element with respect to $*$,
- iv) every $x \in \mathbb{R}$ has an inverse with respect to $*$.

exercise 3. (*8 points*) Let A be the two-element set $A = \{a, b\}$. Write a table of all 16 possible operations on A using the format explained in the book on page 20. Label these operations Op_1 to Op_{16} . Then

- a) among these operations, identify those that are commutative,
- b) identify the operations that have an identity element,
- c) identify the operations for which each element has an inverse.

Note: Example solutions to similar problems can be found at the end of Chapter 2 of the textbook.
